**Pythagorean Scale Length Triangle**

**Geometric Method for Fret Space Calculation via CADD**

Computer-aided design (CAD) is the use of [computer](http://en.wikipedia.org/wiki/Computer) systems to assist in the creation, modification, analysis, or optimization of a [design](http://en.wikipedia.org/wiki/Design). CAD software is used to increase the productivity of the designer, improve the quality of design, improve communications through documentation, and to create a database for manufacturing. CAD output is often in the [form of electronic files](http://en.wikipedia.org/wiki/List_of_file_formats#Computer-aided) for 3D printing, CNC machining, or other manufacturing operations.

CAD software for mechanical design uses either vector-based graphics (X/Y/Z data) to depict the objects of traditional drafting, or may also produce [raster graphics](http://en.wikipedia.org/wiki/Raster_graphics) (Pictures and images) showing the overall appearance of designed objects. As in the manual [drafting](http://en.wiktionary.org/wiki/drafting) of [technical](http://en.wikipedia.org/wiki/Technical_drawing)  [drawings](http://en.wikipedia.org/wiki/Engineering_drawing), the output of CAD must convey information, such as materials, processes, [dimensions](http://en.wikipedia.org/wiki/Dimension), and [tolerances](http://en.wikipedia.org/wiki/Engineering_tolerance), according to application-specific standards (see, ASME Y14.5).

CAD is an important industrial tool widely used in many industries, including commercial products, automotive, boat building, and aerospace industries, mechanical/industrial and architectural design, BioTech, and many more. CAD is also widely used to produce [computer animation](http://en.wikipedia.org/wiki/Computer_animation) for [special effects](http://en.wikipedia.org/wiki/Special_effect) in movies, [advertising](http://en.wikipedia.org/wiki/Advertising) and technical manuals, often called DCC [Digital content creation](http://en.wikipedia.org/wiki/Digital_content_creation).

Computer-aided manufacturing (CAM) is the use of computer software to control [machine tools](http://en.wikipedia.org/wiki/Machine_tool) and related machinery in the [manufacturing](http://en.wikipedia.org/wiki/Manufacturing) of workpieces. CAM may also refer to the use of a computer to assist in all operations of a manufacturing plant, including planning, management, transportation and inventory control. Its primary purpose is to create a faster production process and to produce components and tooling with more precise dimensions and consistency. CAM is a computer-aided process using information created during design and testing of a [computer-aided design](http://en.wikipedia.org/wiki/Computer-aided_design) (CAD) model. The model generated in CAD can be uploaded into CAM software, which then produces codes to control the machine tool.

Traditionally, CAM has been considered as a [numerical control](http://en.wikipedia.org/wiki/Numerical_control) (CNC) programming tool, where in two-dimensional (2-D) or three-dimensional (3-D) models of components generated in [CAD](http://en.wikipedia.org/wiki/Computer_aided_design) software are used to generate [G-code](http://en.wikipedia.org/wiki/G-code) or [M-code](http://en.wikipedia.org/wiki/M-code) etc., which may be company/controller specific, to drive computer numerically controlled ([CNC](http://en.wikipedia.org/wiki/CNC)) machines.

As with other “Computer-Aided” technologies, CAM does not eliminate the need for skilled professionals such as [manufacturing engineers](http://en.wikipedia.org/wiki/Manufacturing_engineer), NC programmers, or [machinists](http://en.wikipedia.org/wiki/Machinist). CAM, in fact, leverages both the value of the most skilled manufacturing professionals through advanced productivity tools, while building the skills of new professionals through visualization, simulation and optimization tools.

**Learning Objectives**

1. The students will calculate fret positions and locations.
2. The students will use the “Tangent Line Method” also known as the “Pythagorean Scale Length Triangle” for laying outing the fret line locations for any given scale length.
3. Students will review the Rule of 18 for determining the distance needed to locate and mark the 1st Fret.
4. Students will create a CAD file ie .DXF drawing that accurately places the locations for all desired frets to any given scale length within .00001 tolerance.
5. Student will use hand tools (calibers, protectors, and bench rules) to measure an existing fret board size and shape.
6. Students will develop a CAD file for a new Fret Board that includes its size, shape, location of fret lines, fret dot markers and any desired inlay.

**Activity #1**

Before beginning this CAD activity, it is recommended that one watches the short video

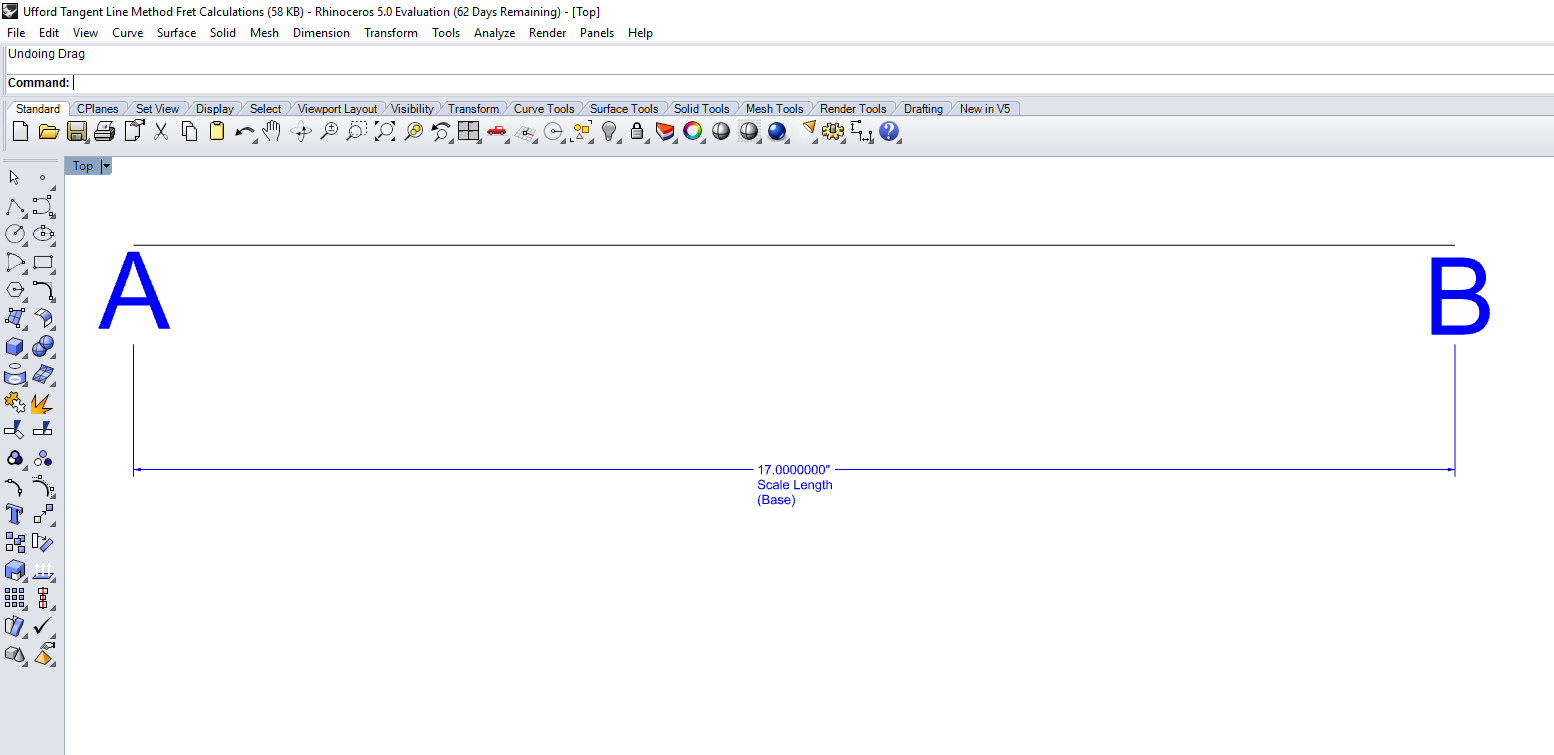
[https://www.youtube.com/watch?v=\_lp-KclqVgQ](https://www.youtube.com/watch?v=_lp-KclqVgQ%20) How to calculate the fret slot positions using the tangent line method. The author of the video calls it a Tangent Line Method, yet with the power of CAD software and infinite zooming capabilities the referred to line it is very close to being a “Tangent” line, but technically not. As a **tangent line** is a straight **line** that touches a function (arc, circle, ellipse, etc..) at only one point. When we zoom in, the “tangent” line is passing through two points that are extremely close together, 0.001470”. Spelled out that is one thousand, four hundred seventeen millionths of an inch. For the sake of an argument, is that amount worth quarrelling over? You decide.

For this activity, we are going to start by drawing a very large and exact triangle via our CAD software. For this example, I am using the CAD software package called Rhinoceros 3D. It is a professional grade high powered 3D Non-uniform rational Basis spline (NURBS) modeler, that also does a fantastic job at flat two dimensional drawings. It is critical that one sets their drawing precision to a minimum of 5 or more decimal places. Also, one needs to use their drawing aids such as Osnaps to locate exact end points, center points, intersections and perpendicular lines.

The base of the triangle will be known as the scale length. The height of the triangle will be known as the Nut Line. And finally, the hypogenous will be known as the Tangent Main Line. Below Steps one and two are the most critical and hinge pin measurements. Get either of these two wrong and you will fail.

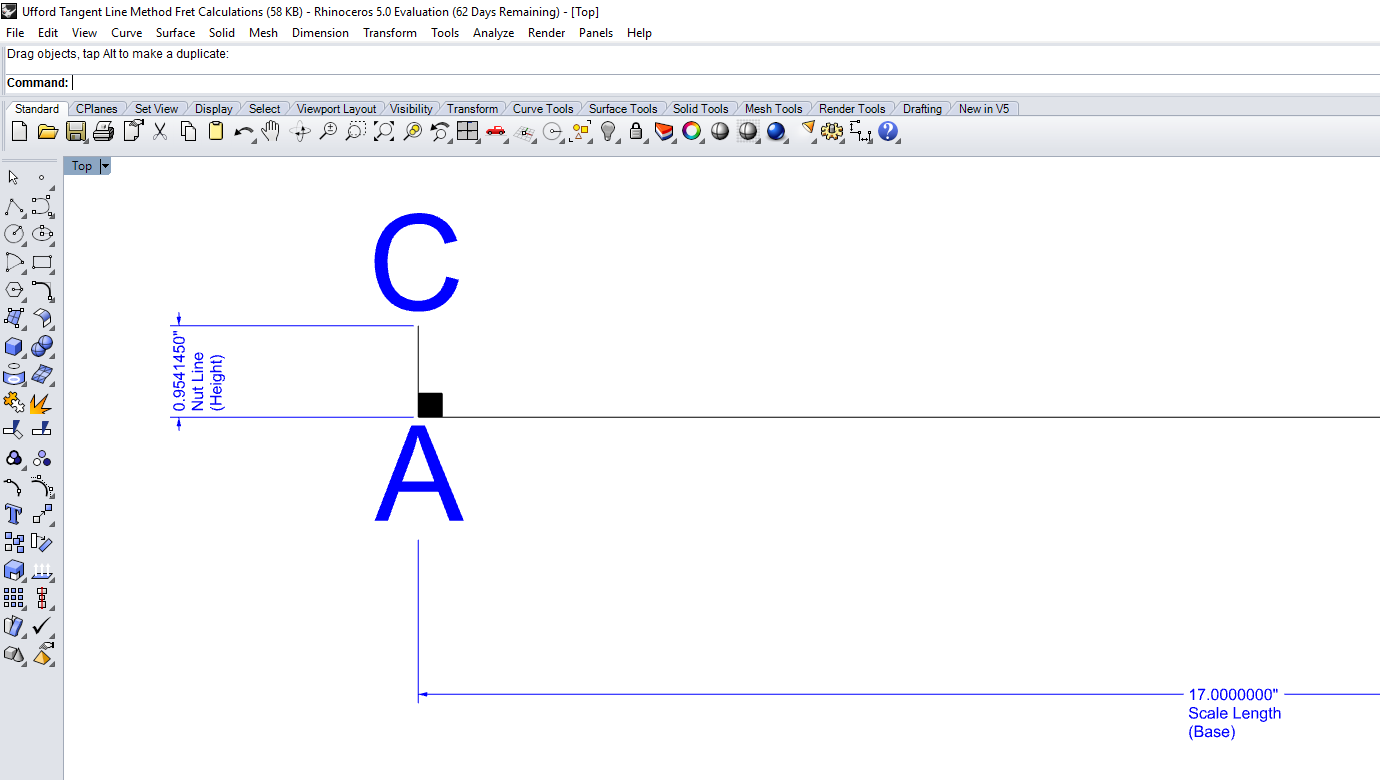
We are going to use the scale length of 17.000” (standard tenor ukulele). But this method will work for any scale length. Also, an accuracy check will be placed at the 12th Fret. It is located exactly one half of the 17.000” scale length (8.500” for our example). We will be marking up to 18 fret lines for this activity.

* Step 1 Draw the base line AB for our triangle. AB is equal to the desired scale length. At point A of line AB will be where the Nut Line Height begins. At point B of line AB is where the Hypotenuse “Tangent Main Line” will eventually intersect. *For the guitar-minded people we can designate the left as fret zero point A, and the right end for the saddle point B.*

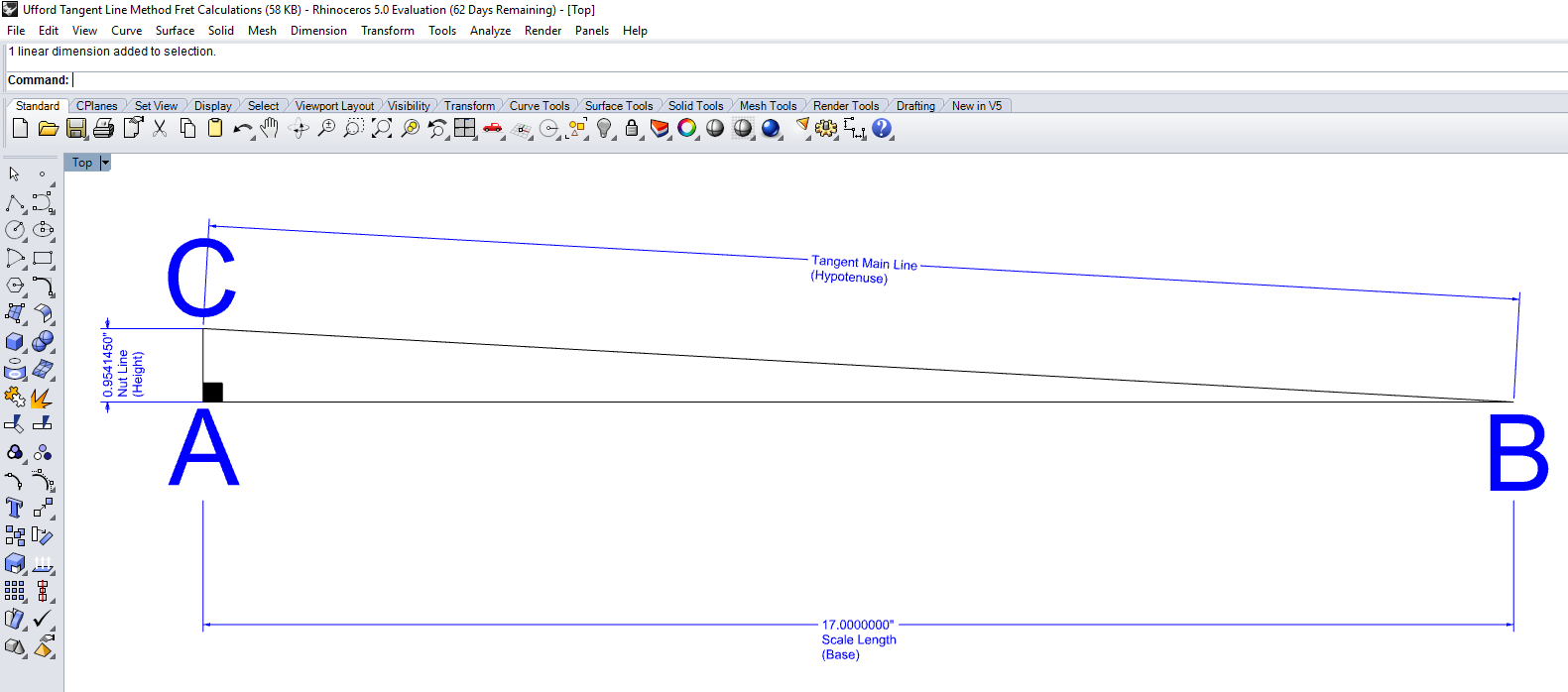


* Step 2 Calculate and draw a line AC. It is equal to the triangle’s height. Before we can find the height of our triangle also known as the “nut line” we must first calculate the distant needed to locate the first fret; Consequently, this is also the same amount for the nut line height.

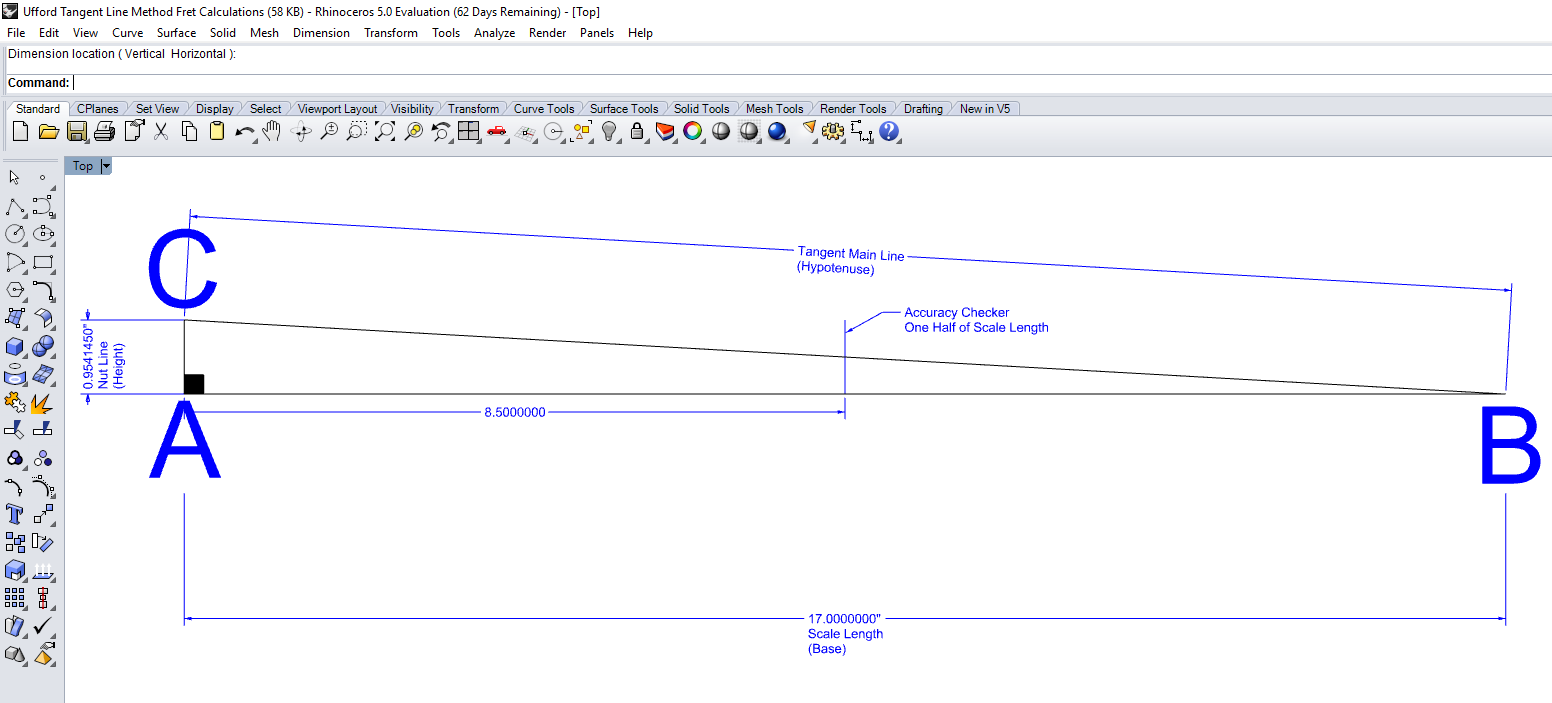
So, using the refined “Rule of 18” (17.817), divide the scale length by 17.817. For example, 17” / 17.817 = 0.95414491777”. The result is the exact distance needed to locate the 1st Fret from point A. As stated earlier it is also the same height of the nut line. Therefore, from point A on base line AB, draw upwards a perpendicular line. The end of this line will be labeled point C.



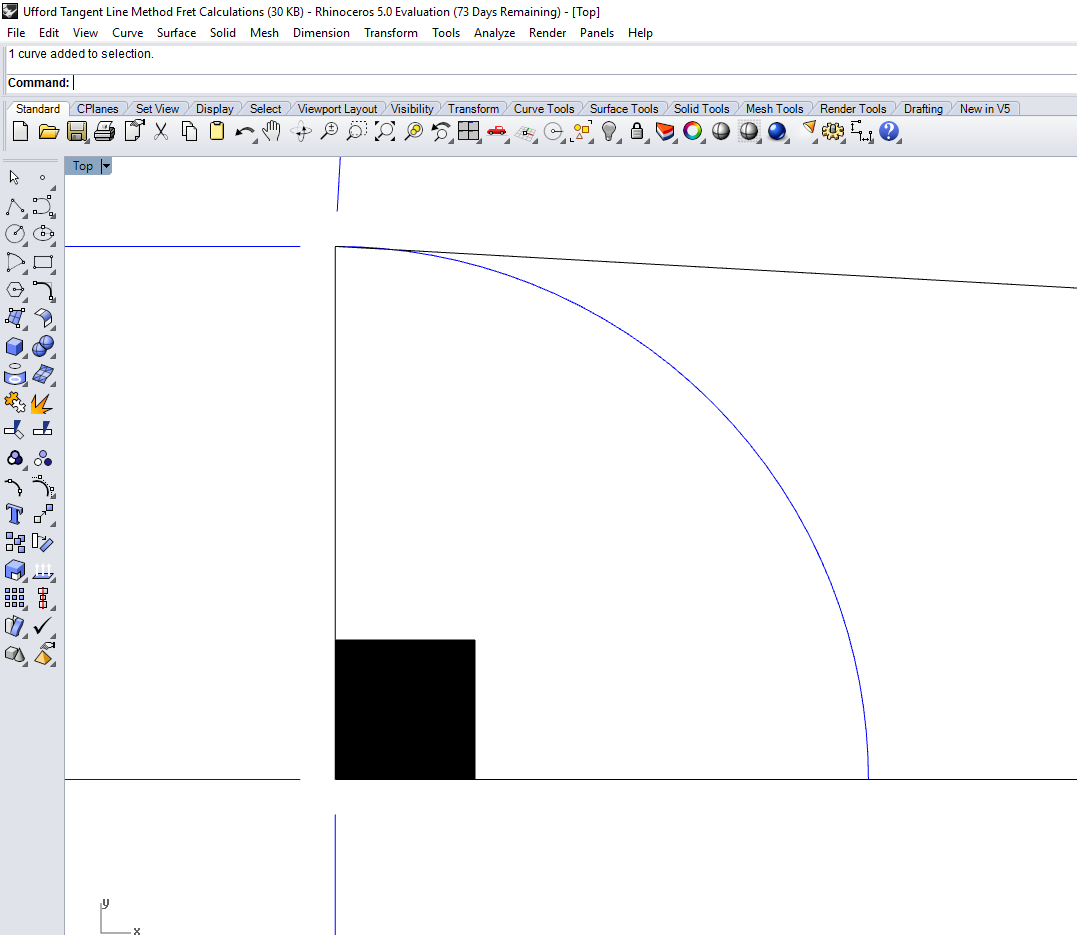
* Step 3 Draw the hypotonus “Tangent Main Line” by connecting points C and B with a straight line. You'll end up with a long triangle as shown here. Congratulations, you have just setup a Pythagorean Scale Length Triangle for a tenor ukulele.



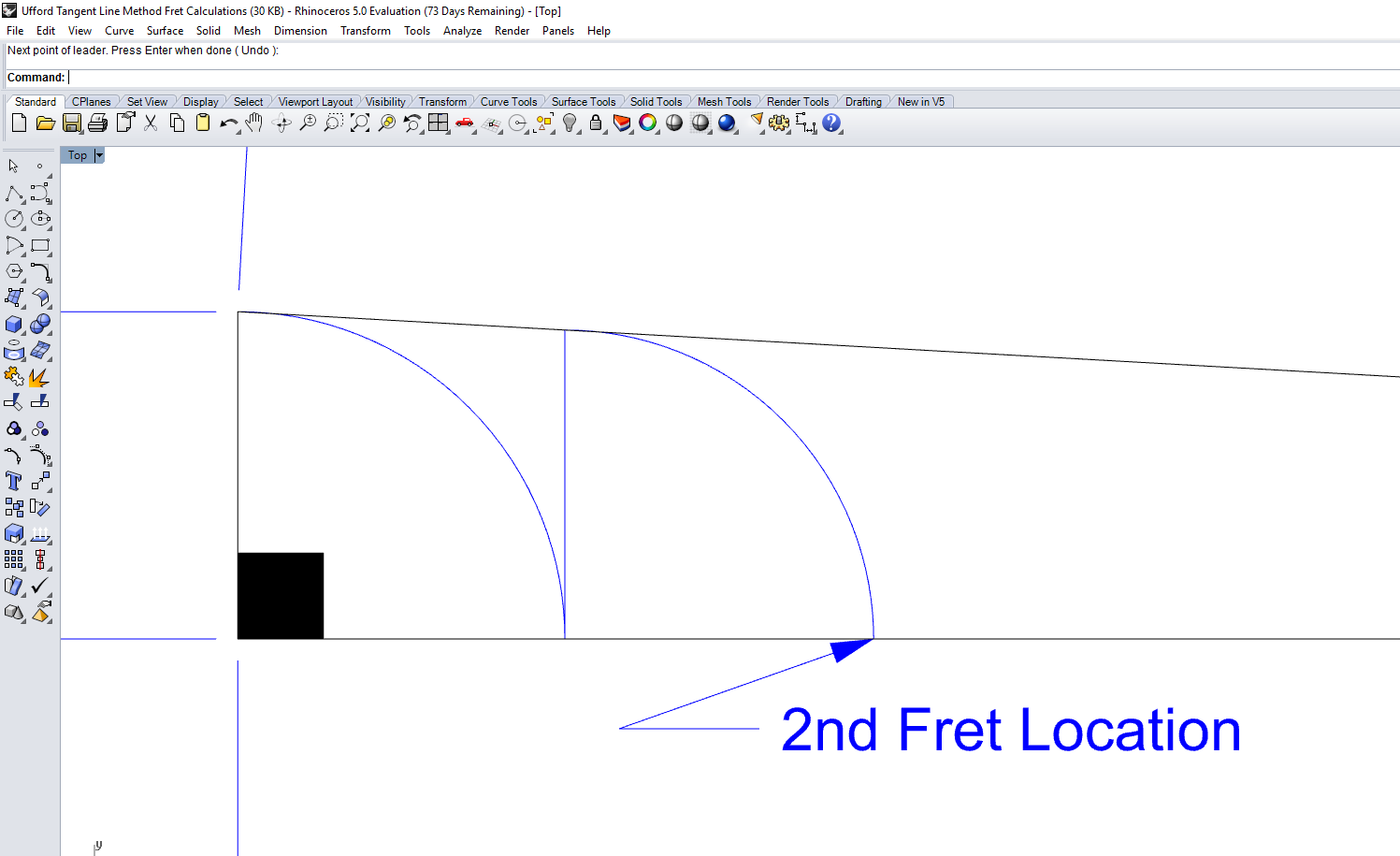
* Step 4 Before we go any further, we are going to place an accuracy checker onto our base line. Bisect base line AB and then draw a short Perpendicular line upwards. You have just placed the accuracy checker. The 12th fret must land exactly at this intersection on the base line AB. If it doesn’t, then you have made an error.

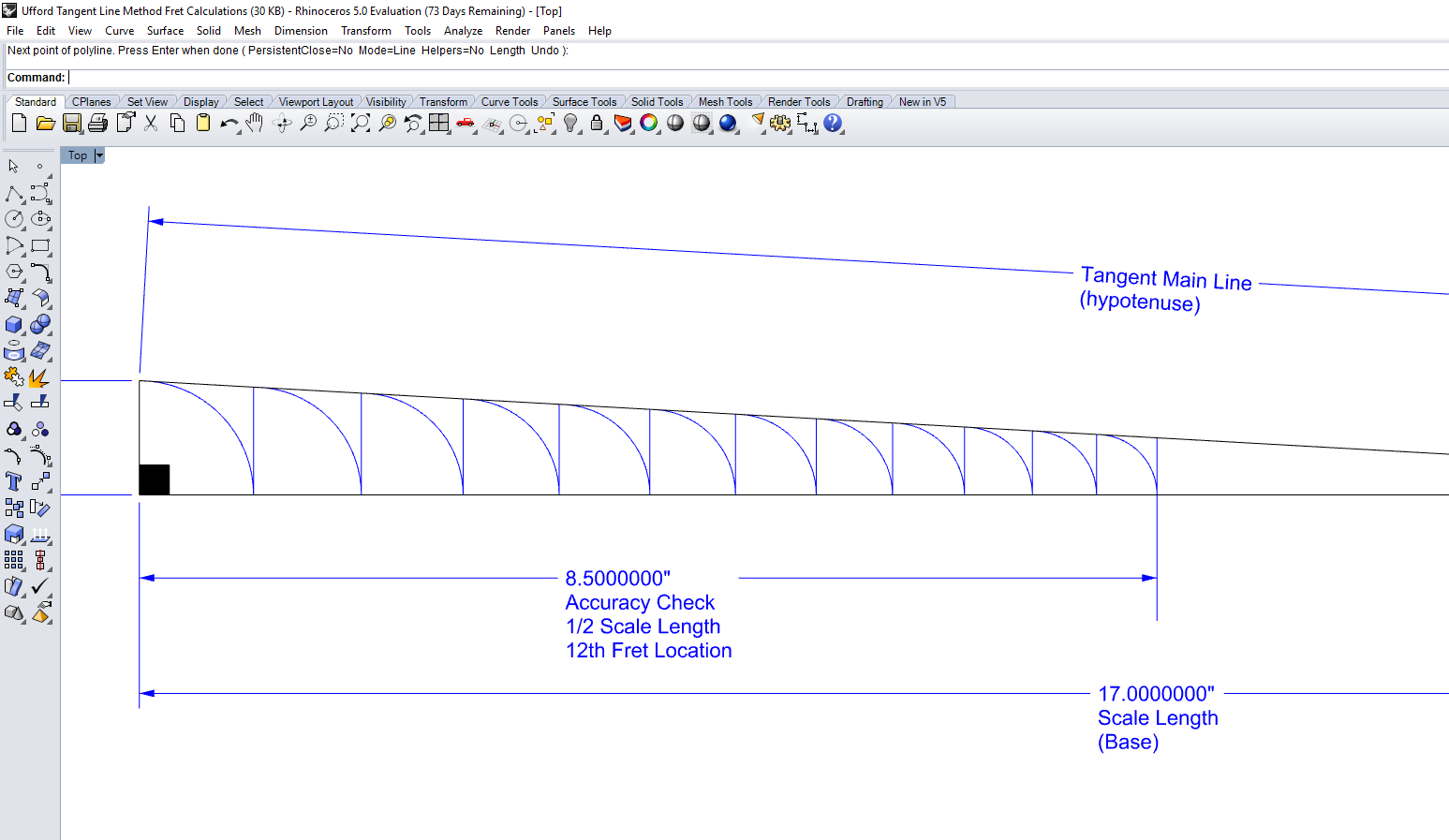
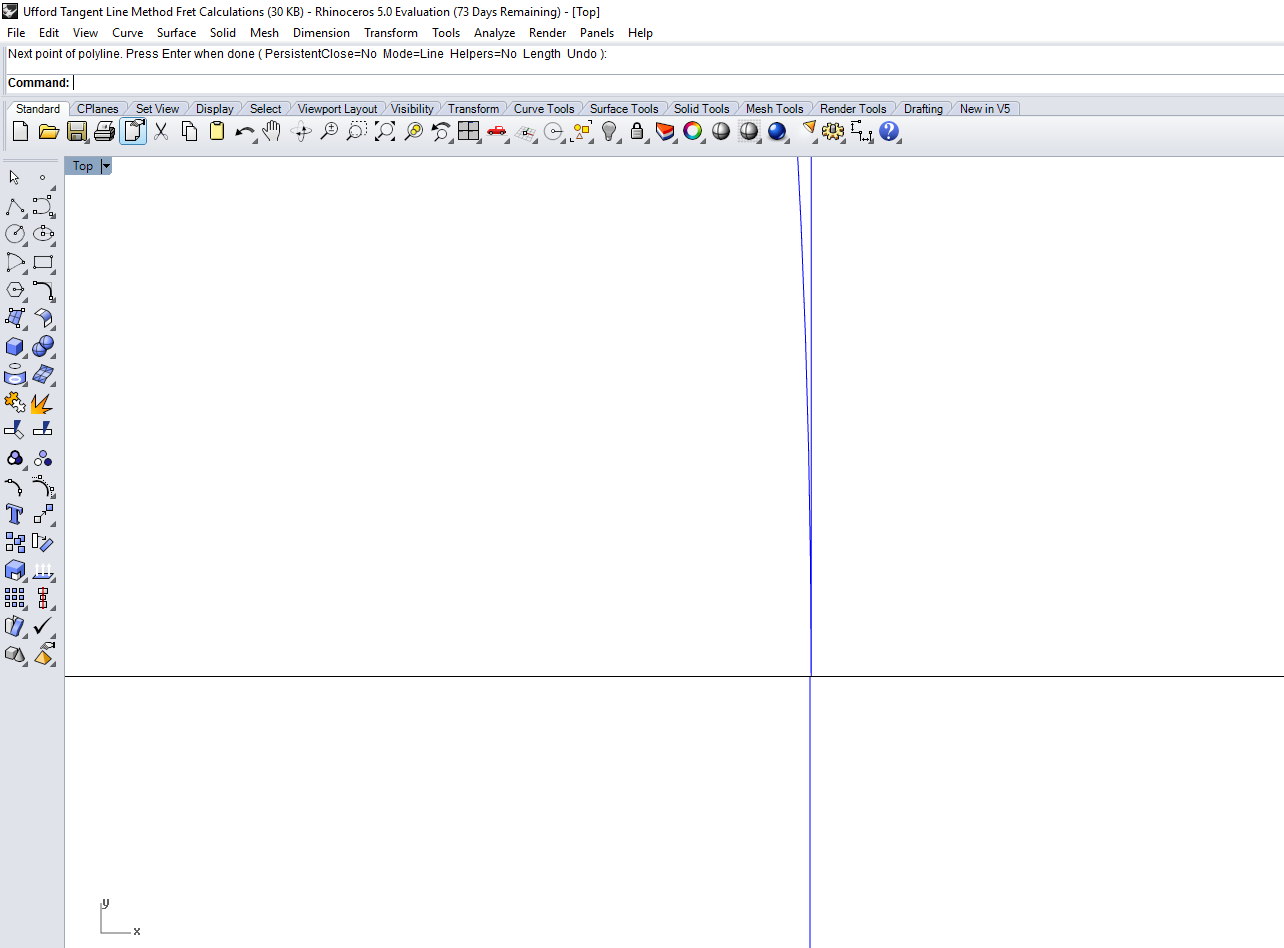
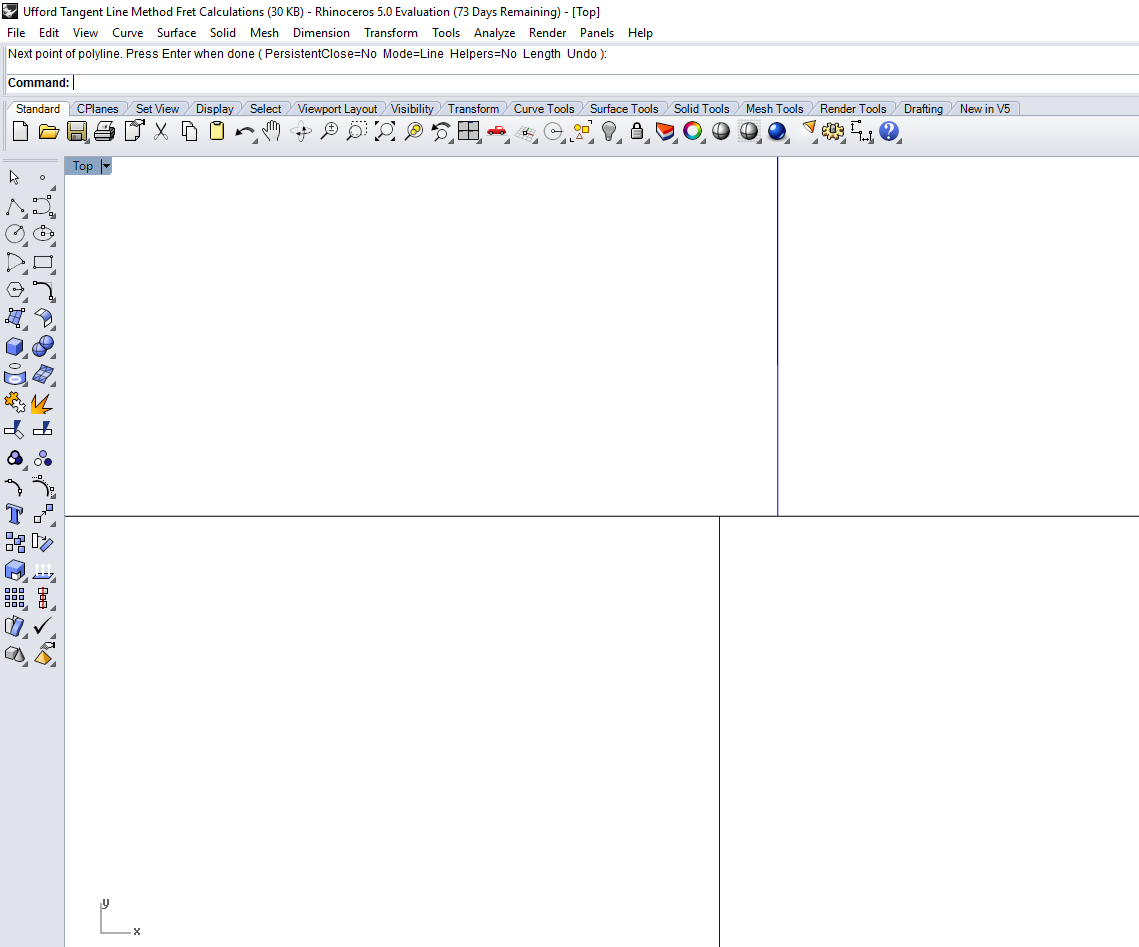
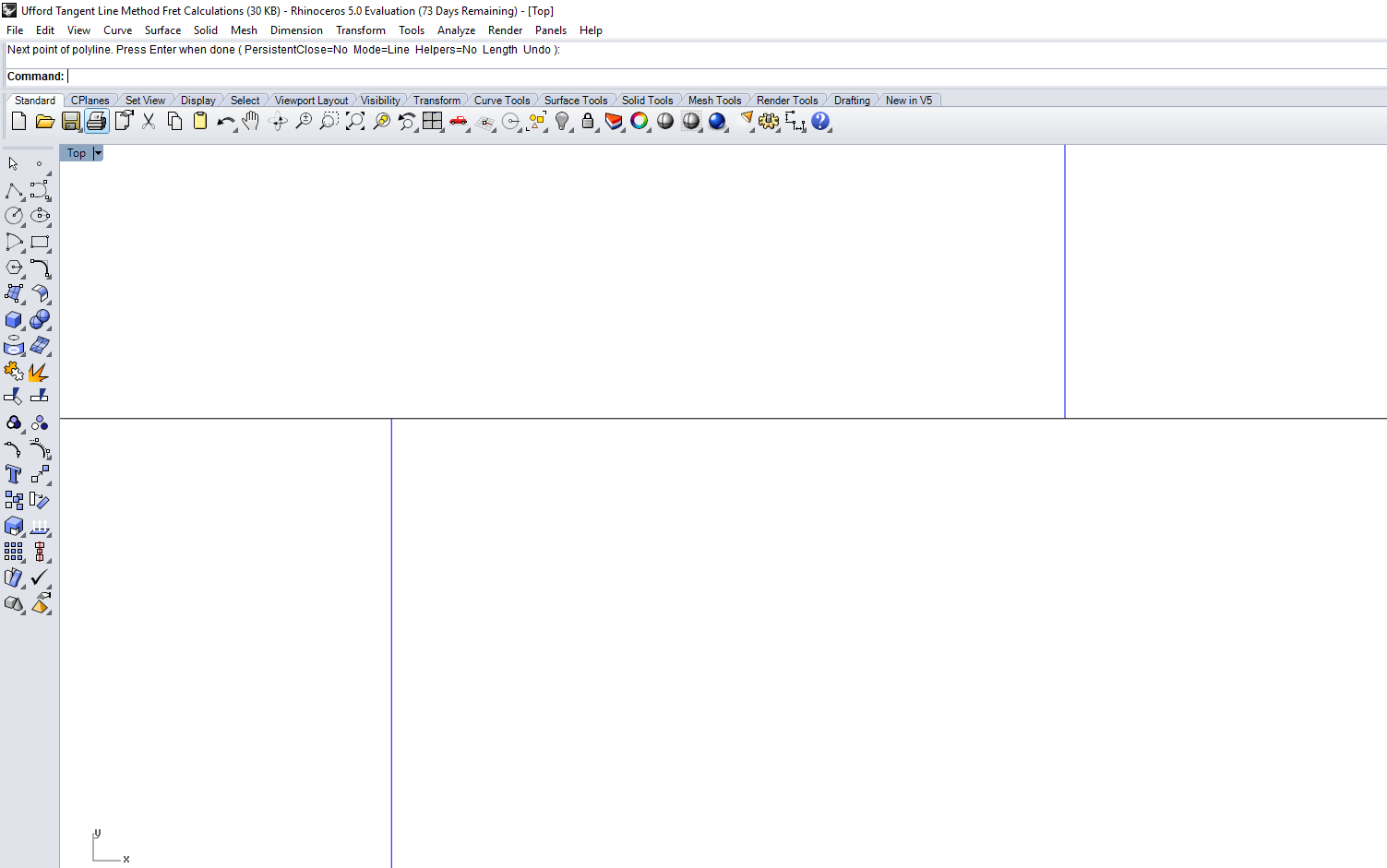


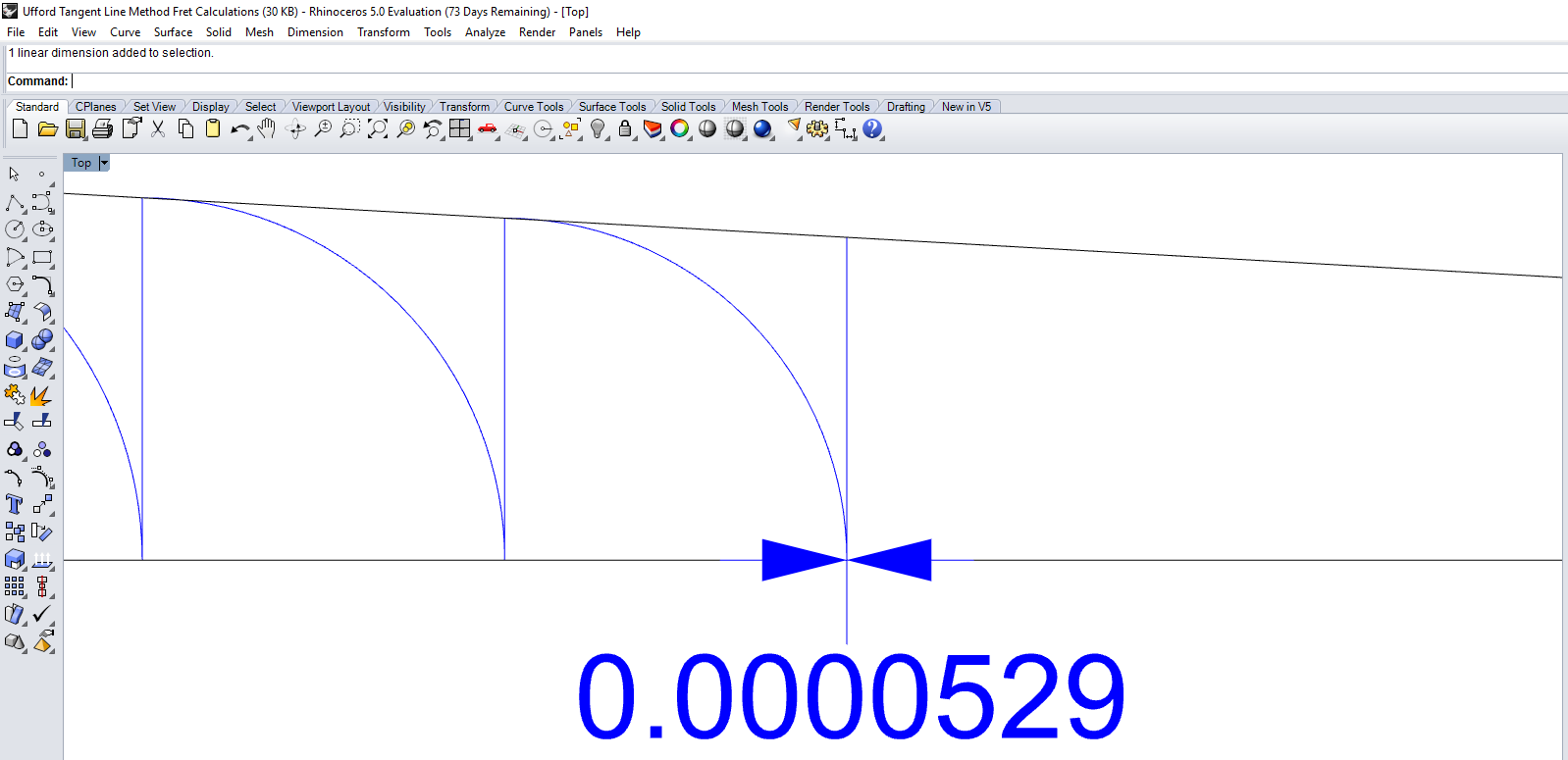
* Step 5 Locate Fret position #1. Using the Arc command, start an arc starting at point A, that spans line AC. Swing/Draw the Arc so it intersects line AB. This intersection point on Line AB is the exact desistance and location for Fret #1.

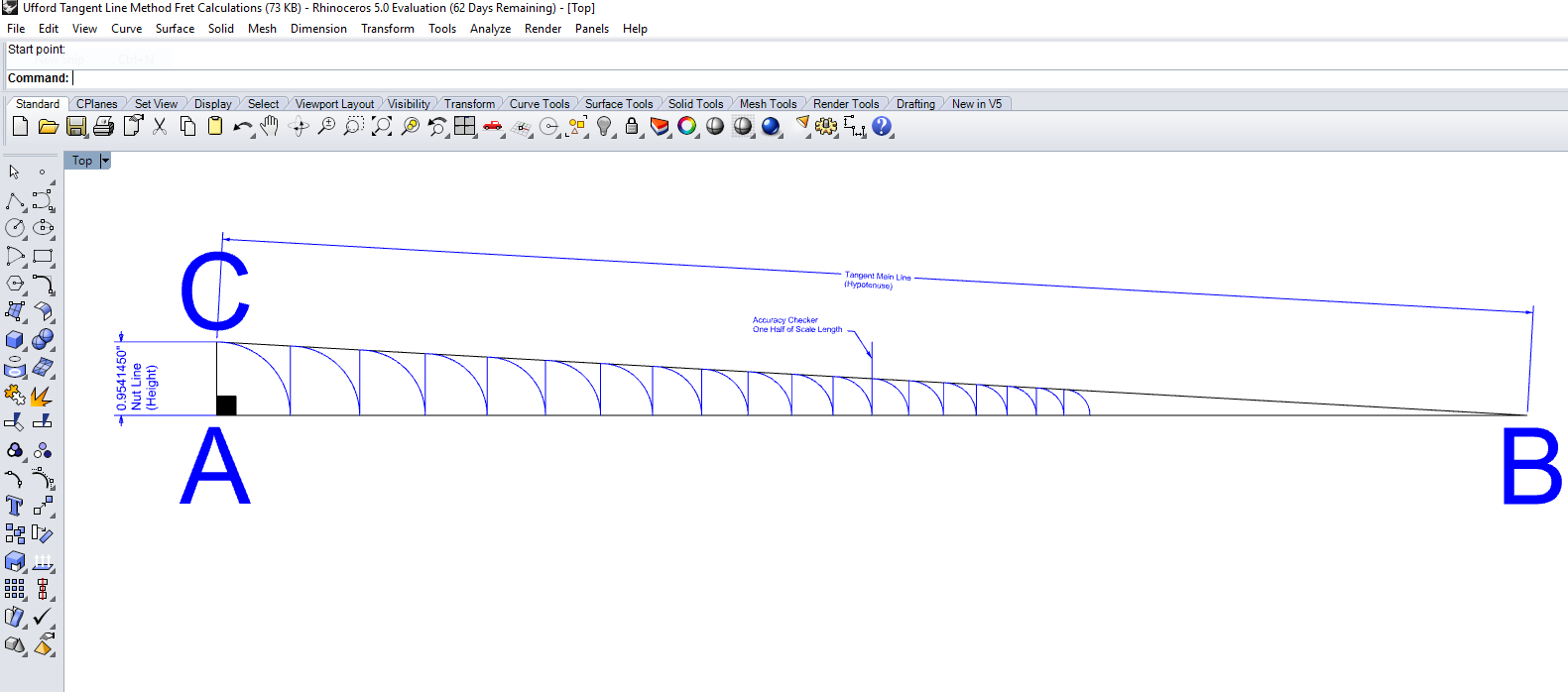


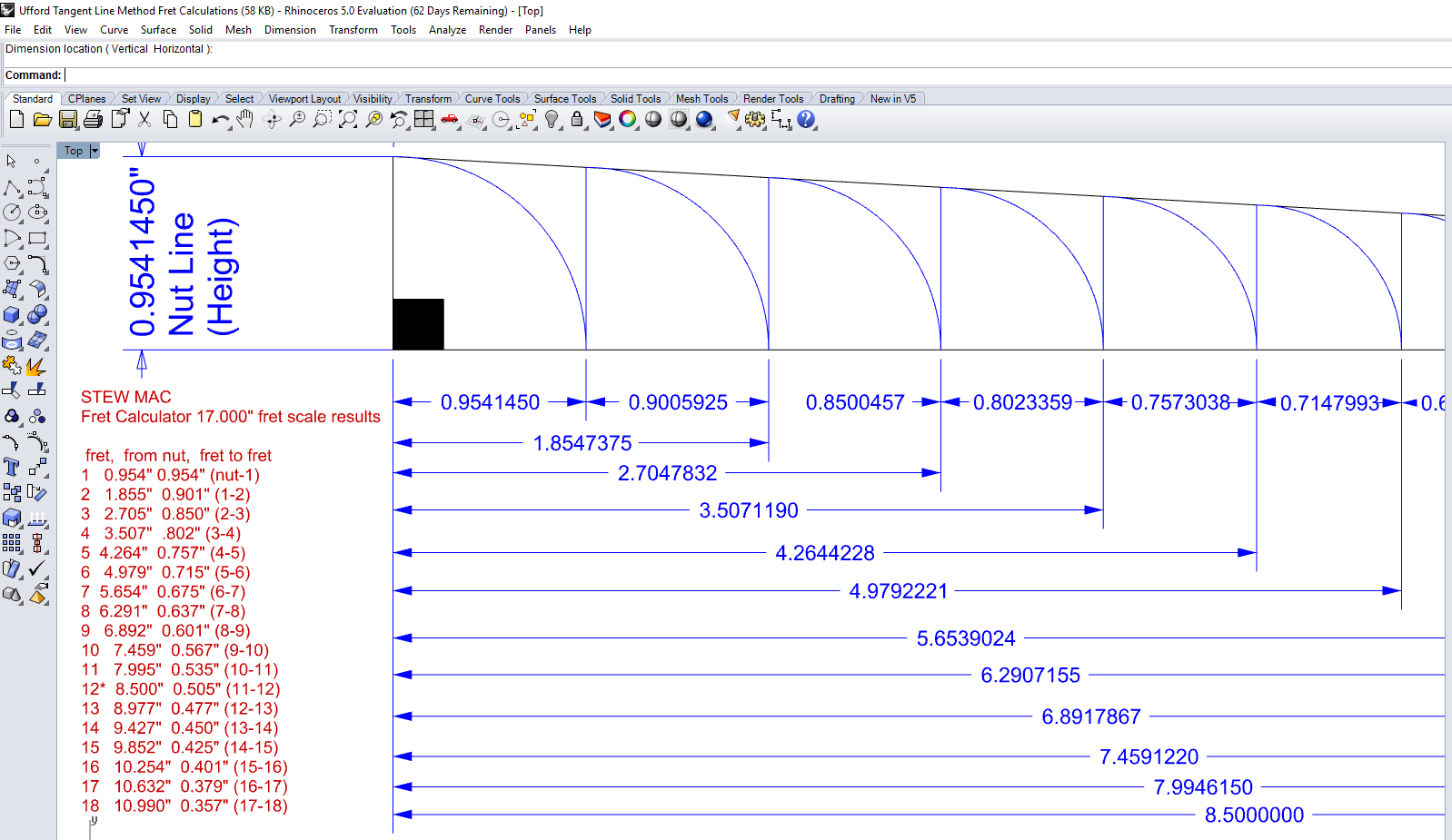
* Step 6 Locate fret position #2. Draw a perpendicular line upwards to intersect the hypotenuse line from Fret #1 location. Now Swing/Draw a new Arc this time starting at Fret #1 that spans the new perpendicular line height. Swing the Arc so it intersects Line AB. You have just located the 2nd Fret position.



* Step 7 Repeat the actions in Step #6 to locate the position of frets #3 thru #12.
* Step 8 Zoom into the 12th Fret location for an Accuracy Check (100x zoom).
* Step 8b Zoom 10,000x magnification
* Step 8c Zoom 100,000x magnification. Use your Dimensioning tools to check the accuracy.
* Step 9 Use the Dimension command to check the accuracy of where the 12th fret should be in relationship of where the Tangent line method located it. For this example, we discovered that we are off by 0.0000529”. **Spelled out that is “five hundred twenty-nine ten-millionths of an inch”.** Question is: Is that an acceptable tolerance for locating fret positions?



* Step 10 Locate the remaining frets. One could continue from there (being off by five hundred twenty-nine ten-millionths of an inch), or simply move over to the true 12th fret location and restart the Tangent Line Method.
* Step 11 Dimension and Compare all fret locations with Stew Mac’s Fret Calculator.



* Step 12 Save your CAD file. In exercise #2 we shall be transforming the fret locating into an actual tenor ukulele fret board that our laser cutter will mark the needed locations and cut out.

**Materials Required**

1. Safety instruction of machine tool operation and setup.

2. CAD/CAM software (Rhino 3D, Autocad, Inventor, Solidworks,) sketch pad and pencil.

3. Measuring tools, such as rulers, decimal equivalents charts, dial calipers, micrometers, calculators.

4. Examples of Tenor ukulele fret boards and any other parts for the data and design plus the application intent of producing a tenor ukulele fret boards.

**References**

* [https://www.youtube.com/watch?v=\_lp-KclqVgQ](https://www.youtube.com/watch?v=_lp-KclqVgQ%20) How to calculate the fret slot positions using the tangent line method. Learn more about my videos at [www.hotstringsguitar.com](file:///C:\Users\Admin\Dropbox\2016%20HHS%20Guitar%20Building\www.hotstringsguitar.com) <http://wwnew.industrialpress.com/products/category_feature/MH> - Machinery’s Handbook
* <http://www.engineeringtoolbox.com/machinability-metals-d_1450.html> - Machinability of metals
* <http://en.wikipedia.org/wiki/Computer-aided_manufacturing>
* <http://en.wikipedia.org/wiki/Computer-aided_design>
* <http://www.featurecam.com>
* http://www.mastercam.com

**Standards**

Standards:

[CCSS.Math.Content.HSF-IF.C.7e](http://www.corestandards.org/Math/Content/HSF/IF/C/7/e) Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

[CCSS.Math.Content.HSF-LE.A.2](http://www.corestandards.org/Math/Content/HSF/LE/A/2) Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).