Trigonometry Supplemental Material for Biology Majors

This manual is the result of collaborative efforts between faculty from the Dept. of Mathematics and Life Sciences at Los Angeles Mission College and Mathematics faculty at University of California, Los Angeles

This manual is best used in conjunction with Academic Excellence Workshop manual.
Website: https://bit.ly/2QCe0ld

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Trigonometry Supplemental Material for Biology Majors

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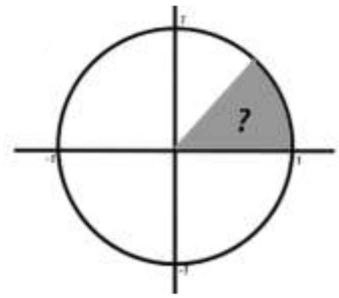
Discovering Radians Activity

Supplies:



- Pre-cut circle from construction paper, or a paper plate
- String or heavy thread
- Ruler and protractor
- Scissors

Steps:

1. Using the string, measure the circumference of the circle (or paper plate). Record this length.

Circumference = ______________

2. Fold the circle (or paper plate) in half. Crease the fold line so that it can be clearly seen.

3. Fold the circle again into quarters. Crease the fold lines.

4. Open the circle. Using the ruler, draw line segments along the fold lines forming four quadrants. Label the points on the edge of the circle that correspond to 0º, 90º, 180º and 270º.

5. The folding process has located the center of the circle. Use your string to measure the radius of the circle. Cut the string to this length. Record this length.

Radius = ___________

6. Hold one end of the radius length string at the edge of the circle at 0º. Wrap the string around the edge of the circle and mark its ending location. Connect this point to the center of the circle.

7. Using your protractor, find the number of degrees in the central angle formed from 0º to the segment you drew in step 6. In terms of radians, this angle has a measure of one radian. Record this answer.

1 Radian = __________˚

8. Using your radius length string, continue to wrap the string around the edge of the circle marking its ending locations. Record the number of radian angles that will fit in the circle.

How many radian angles are in your circle? ___________

9. Ponder:

If the central angle has a radian measure of 2π, what is the number of degrees in the angle?

2π radians = __________˚

Conversion between degrees and radian

To convert an angle measured in degrees to radians, multiply by $\frac{\pi}{180}$.

To convert an angle measured in radians to degrees, multiply by $\frac{180}{\pi}$.

For example:

Ex 1) 225˚=______(radians)

a. To convert to radians you should multiply by ______.
b. You should only focus on ________ and write π after the fraction is reduced.
c. _________ = _________.
d. The answer is ________.

Ex 2) — = ______(degrees)

a. To covert to degrees you should multiply by _______.
b. Since the π's cancel each other, we should only focus on the ________.
c. ________ = _______.
d. The answer is _______.

Ex 3) An angle representing one complete revolution of the unit circle measures 2π radians, formerly ______º.

1. Change the following radians to degrees if 2π=360º,

a) π = ____________

b) $\frac{\pi}{2} =$ ____________

c) $\frac{\pi}{4} =$ ____________

d) $\frac{3\pi}{4} =$ ____________

e) $\frac{11\pi}{6} =$ __________

2. Change the following degrees to radians if 360º= 2π ,

a) 270º = __________

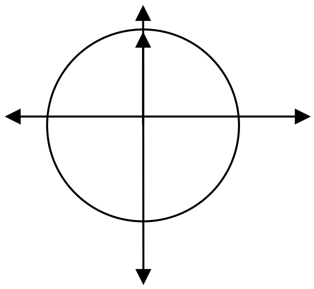
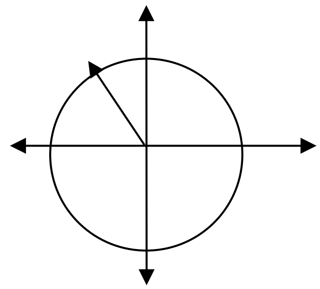
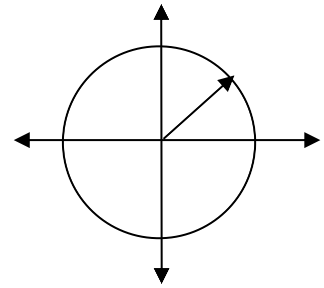
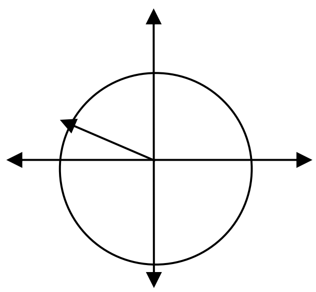
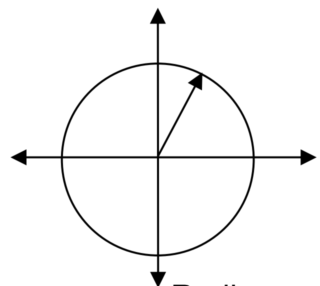
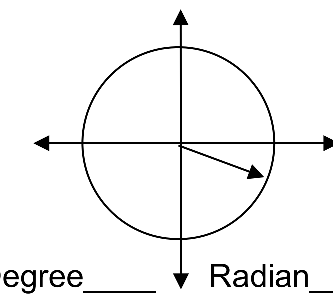
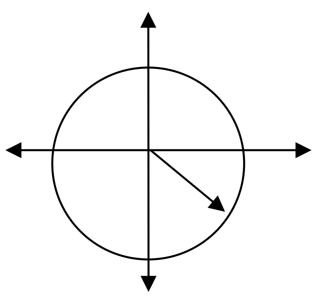
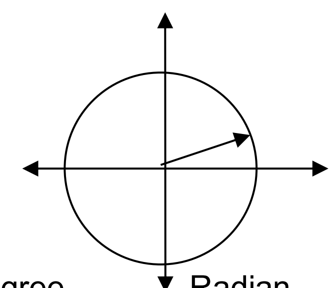
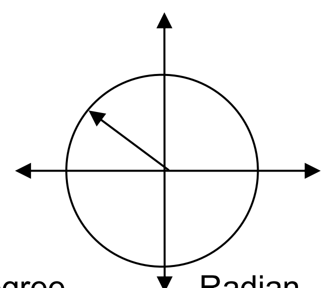
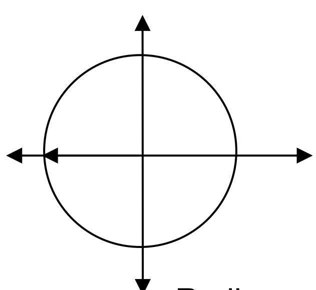
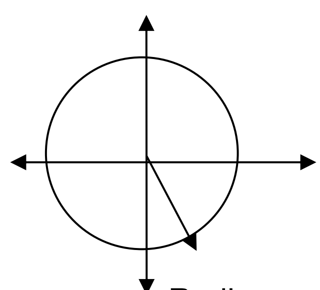
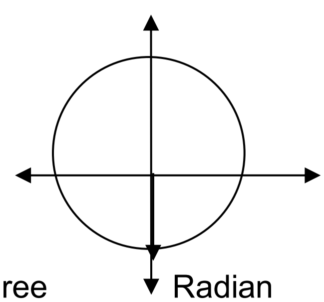
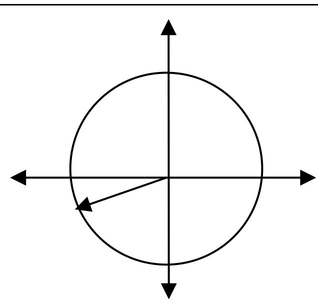
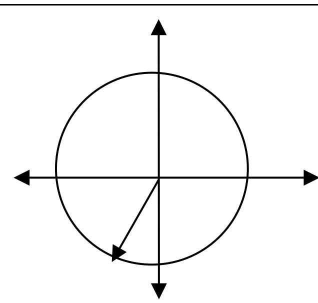
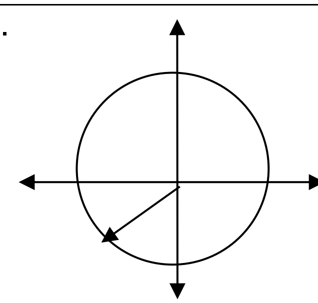
b) 60º = __________

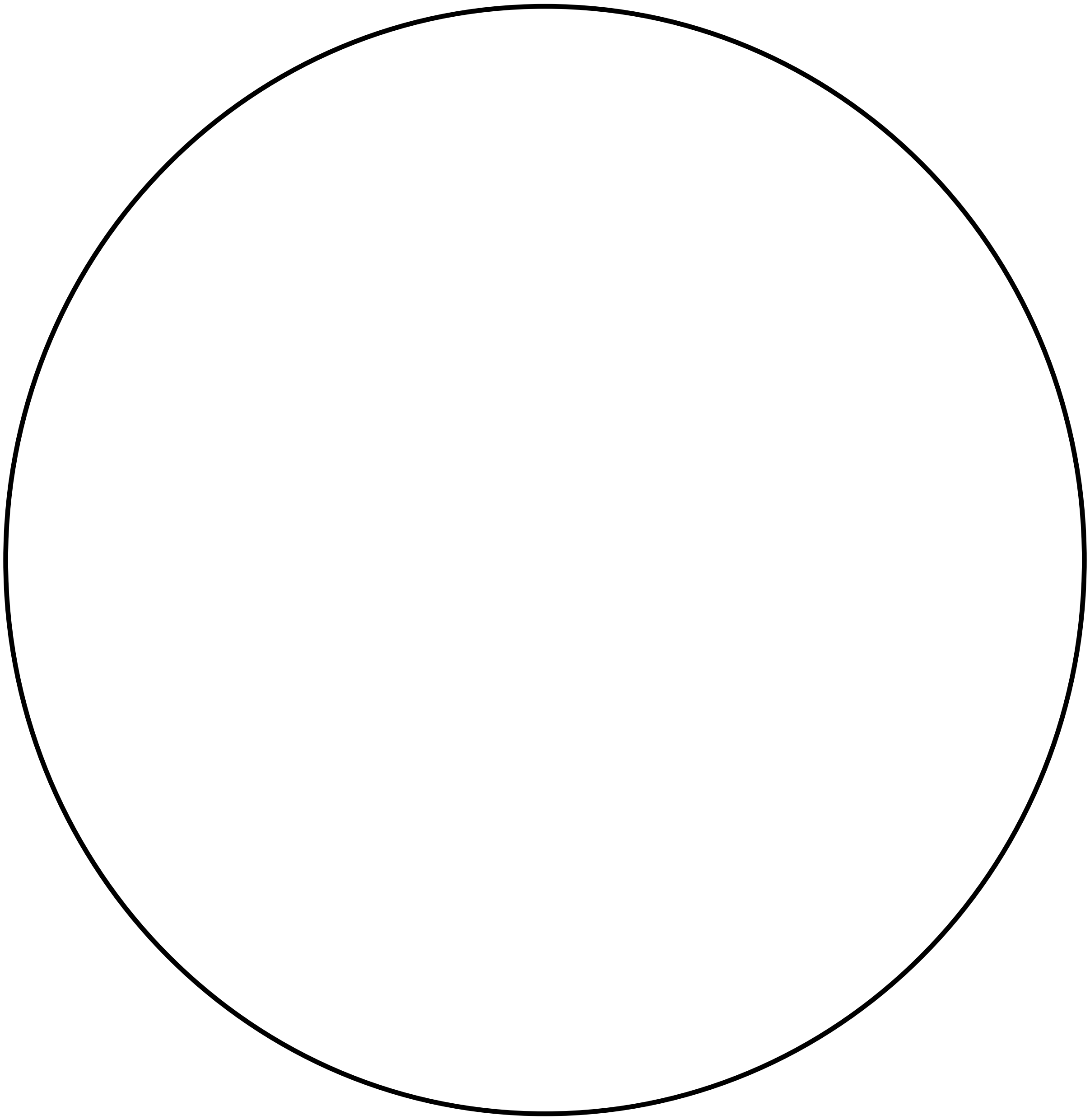
c) 150º = __________

d) 30º = __________

e) 240º = __________

Work with partners to find the angles in degrees <u>and</u> radians.

1.  Degree______ Radian______	2.  Degree_____ Radian_____	3.  Degree_____ Radian_____
4.  Degree_____ Radian_____	5.  Degree_____ Radian_____	6.  Degree_____ Radian_____
7.  Degree_____ Radian_____	8.  Degree_____ Radian_____	9.  Degree_____ Radian_____
10.  Degree_____ Radian_____	11.  Degree_____ Radian_____	12.  Degree_____ Radian_____
13.  Degree_____ Radian_____	14.  Degree_____ Radian_____	15.  Degree_____ Radian_____



Applications of Trigonometric Functions

1. The root system for some native Caribbean plants requires 5 m^2 of land area to collect the required amount of nutrients.
   a. If this land area is circular, what is the area?
   b. If this land is a 35º sector of a circle between two rocks, what is the radius?

2. The equation P = 20 sin(2πt) + 100 models the blood pressure, P, where t represents time in seconds.
   a. Find the blood pressure after 15 seconds.
   b. What are the maximum and minimum blood pressures?

3. The amount of nutrients in plants depends on the amount of sunlight they receive. The amount of sunlight in a certain region can be modeled by the function $h = 15\cos\left(\frac{1}{600}d\right)$, where h represents the hours of sunlight, and d is the day of the year. Use the equation to find how many hours of sunlight there are on February 10, the 42nd day of the year.

4. At Mauna Loa, Hawaii, atmospheric Carbon Dioxide levels in parts per million (ppm) have been measured regularly since 1958. The function defined by $L(x) = .022x^2 + .55x + 316 + 3.5\sin(2\pi x)$ can be used to model levels, where x is in years and $x = 0$ corresponds to 1960.
   a. Calculate the Carbon Dioxide levels in 1970.
   b. Calculate the Carbon Dioxide levels in 2017.

Introduction to Trigonometric Functions and the Unit Circle

1) **Find the angle of smallest possible positive measure coterminal with the given angle:** $-172°$

2) **Find the angle of smallest possible positive measure coterminal with the given angle:** $438°$

3) **Suppose that** θ **is in standard position and the given point is on the terminal side of** θ**. Give the exact value of the indicated trig function for** θ**:** (6, 8); Find $\csc \theta$.

4) **Convert** $330°$ **to radians. Leave answer as a multiple of** π**.**

5) **Convert t** 0.2521 **to degrees. Give answer using decimal degrees to the nearest hundredth. Use 3.1416 for** π**.**

6) **Find the exact value:** $\sin(-180°)$

7) **Find the exact value:** $\sec 270°$

8) **Find the exact value:** $\cot 90° + 2 \cos 180° + 6 \sec^2 360°$

9) **Find the exact value:** $\cos 30°$

10) **Find the exact value:** $\tan \frac{-5\pi}{6}$

11) **Find the exact value:** $\csc \frac{4\pi}{3}$

12) **Find the length of an arc intercepted by a central angle** $\theta = \frac{\pi}{3}$ radians **in a circle of radius** $r = 38.81$ ft; **. Round your answer to 1 decimal place.**

13) **Assume that the cities lie on the same north–south line and that the radius of the earth is 6400 km.** Find the latitude of Spokane, WA if Spokane and Jordan Valley, OR, 43.15° N, are 486 km apart.

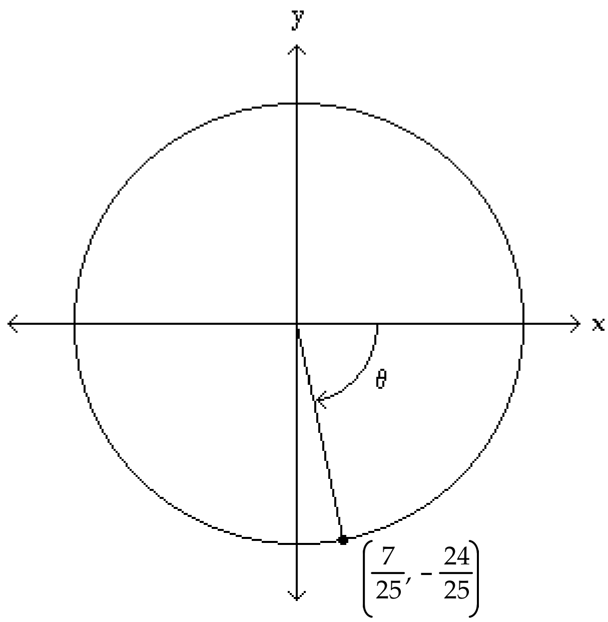
14) **Find the area of a sector of a circle having radius** $r = 15.0$ ft, **and central angle** $\theta = \frac{2\pi}{3}$ radians**. Express the answer to the nearest tenth**

15) Each tire of an automobile has a radius of 2 feet. How many revolutions per minute (rpm) does a tire make when the automobile is traveling at a speed of 79 feet per sec? Round your answer to the nearest tenth.

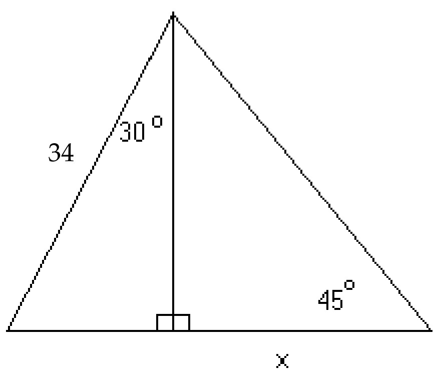
16) Find $\sec \theta$ if $\cos \theta = \frac{1}{4}$ and $\sin \theta > 0$.

17) Find cot θ if csc θ = $\frac{\sqrt{37}}{6}$ and θ is in quadrant I.

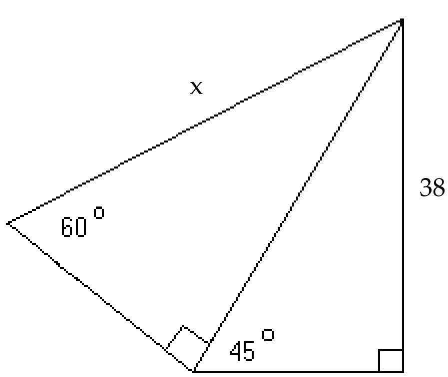
18) Find cot θ.



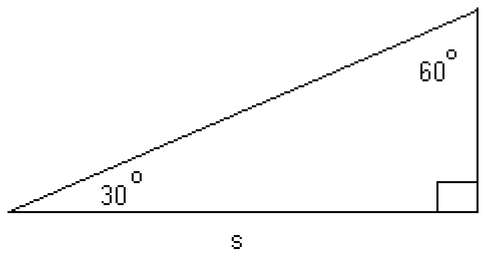
19) Find the exact value of x in the figure.



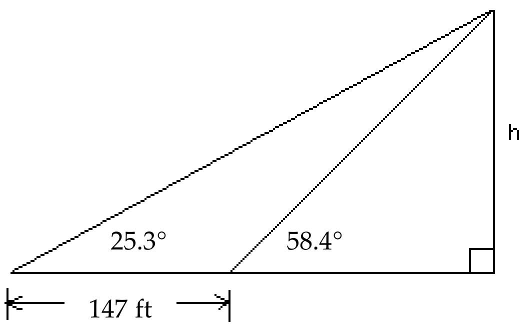
20) Find the exact value of x in the figure.



21) Find a formula for the area of the figure in terms of s.



22) Find h as indicated in the figure. Round your answer to the hundredths place.



23) An airplane travels at 160 km/h for 4 hr in a direction of 306° from St. Louis. At the end of this time, how far west of St. Louis is the plane (to the nearest kilometer)?

24) The angle of elevation from a point on the ground to the top of a tower is 37.87°. The angle of elevation from a point 106 feet farther back from the tower is 24.15°. Find the height of the tower. Round your answer to the hundredths place.

Answer Key

1) 188°
2) 78°
3) $\frac{5}{4}$
4) $\frac{11\pi}{6}$
5) 14.44°
6) 0
7) Undefined
8) 4
9) $\frac{\sqrt{3}}{2}$
10) $\frac{\sqrt{3}}{3}$
11) $-\frac{2\sqrt{3}}{3}$
12) 40.6 ft
13) 47.50 °N
14) 235.6 ft^2
15) 377.2 rpm
16) 4
17) $\frac{1}{6}$
18) $-\frac{7}{24}$
19) $17\sqrt{3}$
20) $\frac{76\sqrt{6}}{3}$
21) $\frac{\sqrt{3}}{6}\ s^2$
22) 97.98 ft
23) 518 km
24) 112.26 ft

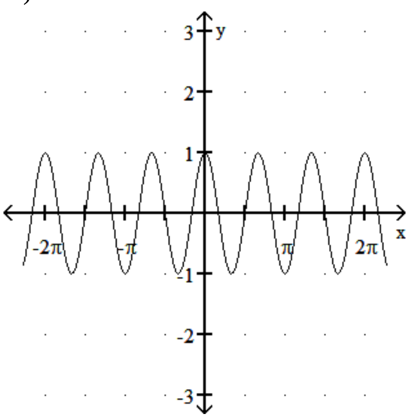
Graphing Trigonometric Functions

For textbook reference you can use the free opnestax Precalculus text: https://openstax.org/details/books/precalculus
Sections 6.1 and 6.2

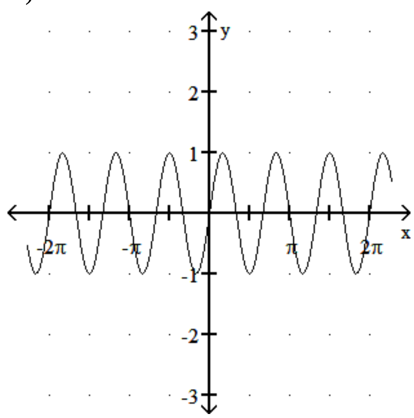
1) **Match the function with its graph.**

1) $y = \sin 3x$ 2) $y = 3 \cos x$
3) $y = 3 \sin x$ 4) $y = \cos 3x$

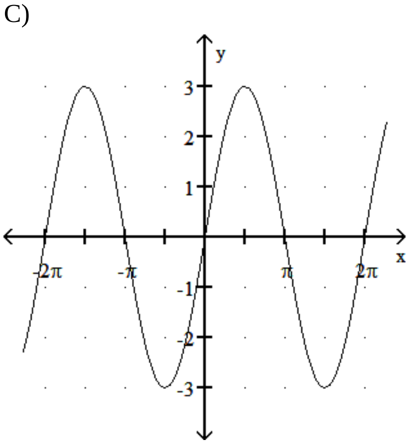
A)



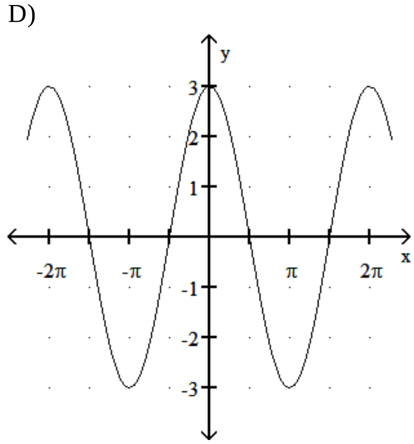
B)



C)



D)



2) **Give the amplitude:** $y = -2 \cos \frac{1}{3}x$

3) **Give the period:** $y = \cos 5x$

4) **Find the phase shift of the function.** $y = -3 + 2 \sin\left(3x - \frac{\pi}{6}\right)$

5) **Graph the function over a one-period interval:** $y = 4 + \frac{1}{3} \sin (2x - \pi)$

6) **Graph the function over a one-period interval.** $y = \frac{1}{2} + \cos\left(2x - \frac{2\pi}{3}\right)$

7) The population size of most insects is dependent on the temperature of their habitats. The temperature in Fairbanks is approximated by $T(x) = 37 \sin\left[\frac{2\pi}{365}(x - 101)\right] + 25$, where T(x) is the temperature on day x, with x = 1 corresponding to Jan. 1 and x = 365 corresponding to Dec. 31. Estimate the temperature on day 10.

8) **Graph the function.:** $y = \frac{4}{5} \tan\left(\frac{1}{2}x - \frac{\pi}{6}\right)$

9) **Graph the function.** $y = -\frac{4}{5} \cot\left(\frac{1}{2}x - \frac{\pi}{6}\right)$

10) **Graph the function.** $y = \csc\left(x - \frac{\pi}{3}\right)$

11) **Graph the function.** $y = 2 + 4 \sec\left(x + \frac{\pi}{5}\right)$

12) The average person's blood pressure is modeled by the function $f(t) = 20 \sin(160\pi t) + 100$, where f(t) represents the blood pressure at time t, measured in minutes.

a) Graph the function.

b) Find the highest and lowest values for the average blood pressure and the time at which they occur.

13) Circadian rhythms are approximately 24-hour internally controlled biological changes that occur in the absence of environmental cues- although they can be altered by the environment. The first example that might pop in your mind when you hear circadian rhythms is the sleep-wake cycle. Other examples include daily fluctuations in fluctuations in body temperature, hormones, behavior, and heart rate. Individual neurons in the suprachiasmatic nucleus, a cluster of cells in the region of the brain called the hypothalamus, generates this "biological clock" in mammals. But you don't need to have a brain to have a biological clock. Circadian rhythms are found in a wide variety of organisms- from single-celled yeast to plants.

For this module, we will consider a hypothetical example. Suppose a particular species exhibits daily regular fluctuations in body temperature that can be approximated by the equation,

$$T(t) = 36.8 - 1.3\sin\left(\frac{\pi}{12}(t + 2)\right)$$

where T represents temperature in°C, t represents time (in hours), and t = 0 corresponds to 12 o'clock midnight (12:00 am)

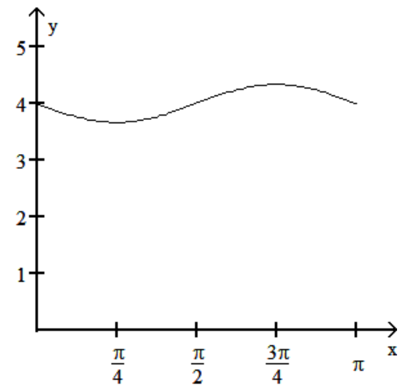
a) Find tte period of the function. Does the period make sense? Why?
b) What tiem of the day, does the body temperature reach the maximum? What is the temperature at that time?
c) Approximate the body temperature at 10 am.

Answer Key
Testname: GRAPHING TRIG FUNCTIONS

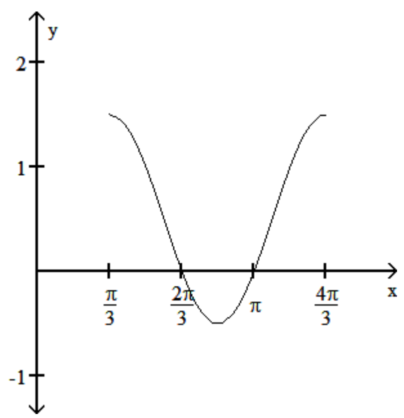
1) 1B, 2D, 3C, 4A
2) 2
3) $\frac{2\pi}{5}$

4) $\frac{\pi}{18}$ units to the right

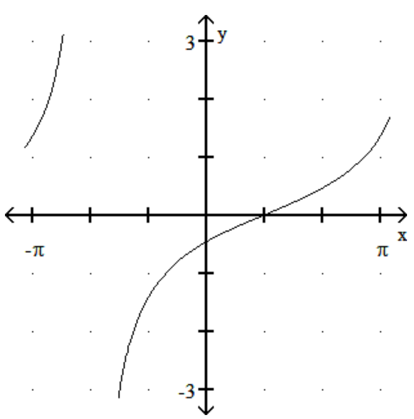
5)



6)



7) -12°
8)



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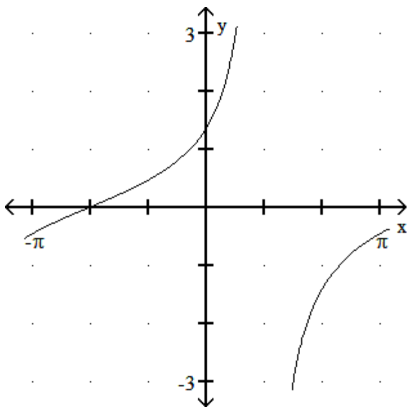

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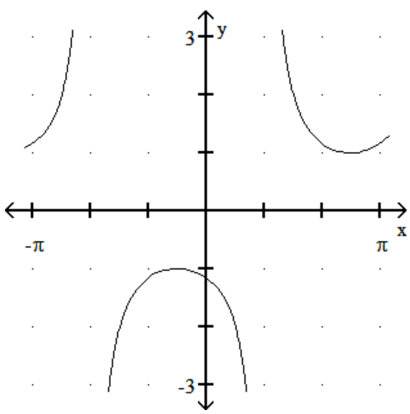

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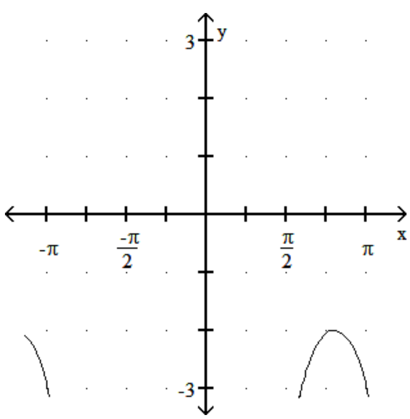

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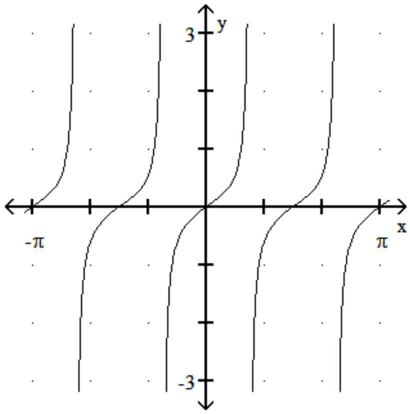


13)

Inverse Trigonometric Functions

For textbook reference you can use the free opnestax Precalculus text: https://openstax.org/details/books/precalculus
Section 6.3

1) Find the exact value of the real number y. $y = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

2) Find the exact value of the real number y. $y = \arctan(1)$

3) Find the exact value of the real number y. $y = \sin^{-1}(0.5)$

4) Graph: $y = \operatorname{arccsc} \frac{1}{3}x$

Evaluate the expression.

5) $\cos\left(\arcsin \frac{1}{4}\right)$

6) $\arccos\left(\cos \frac{\pi}{2}\right)$

7) $\sin(\arctan 2)$

Write the following as an algebraic expression in u, u > 0.

8) $\cos(\arcsin u)$

9) $\tan\left(\cos^{-1} \frac{u}{3}\right)$

10) $\cos(\arctan u)$

11) $\sin\left(\arctan \frac{u}{\sqrt{2}}\right)$

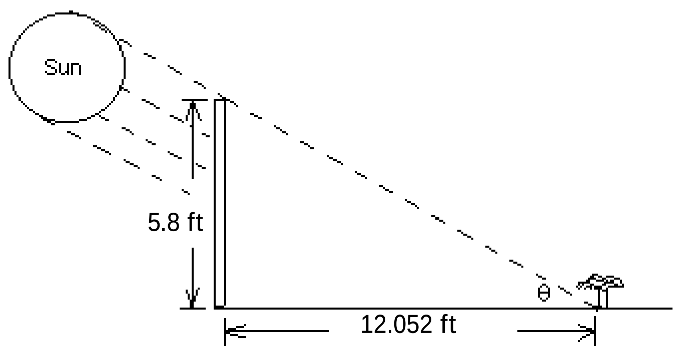
12) True or false? The statement $\cos(\cos^{-1} x) = x$ for all real numbers in the interval $0 \le x \le \pi$.

13) True or false? The statement $\tan^{-1}(\tan x) = x$ for all real numbers in the interval $-\infty \le x \le \infty$.

14) A crystal is an array of atoms that forms atomic layers known as atomic planes. When an x-ray is passed through a crystal, the x-ray beam is diffracted according the crystal's atomic structure. Using a technique called x-ray crystallography, one can construct the three dimensional atomic structure based upon the diffraction pattern.
X-ray crystallography has been used to uncover the atomic structure of thousands of macromolecules ranging from vitamins to protein complexes. X-ray crystallography was a critical technique in many discoveries that were honored with the Nobel Prize. Perhaps the most famous structure revealed by x-ray crystallography is the double helical structure of DNA.
The structure of a crystal can be experimentally determined by Bragg's equation, $n\lambda = 2d \sin\theta$
where λ is the wavelength of x-rays, d is the distance between atomic planes, θ is the angle of reflection (in degrees), and n is a positive integer.

a) Assume that for a given crystal n=1, find the angle of reflection, if the wavelength adn teh distance are equal.
b) Assume that n=2, and the distance is three times the wavelength, use your calculator to approximate the angle.

15) A 5.8-ft fence is 12.052 ft away from a plant in the direction of the sun. It is observed that the shadow of the fence extends exactly to the bottom of the plant. (See drawing) Find θ, the angle of elevation of the sun at that time. Round the measure of the angle to the nearest tenth of a degree.

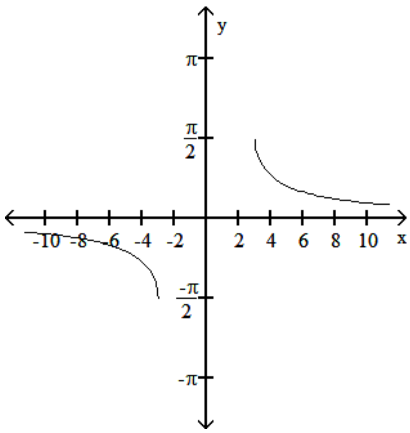


1) $\frac{\pi}{4}$

2) $\frac{\pi}{4}$

3) $\frac{\pi}{6}$

4)



5) $\frac{\sqrt{15}}{4}$

6) $\frac{\pi}{2}$

7) $\frac{2\sqrt{5}}{5}$

8) $\sqrt{1 - u^2}$

9) $\frac{\sqrt{9 - u^2}}{u}$

10) $\frac{\sqrt{u^2 + 1}}{u^2 + 1}$

11) $\frac{u\sqrt{u^2 + 2}}{u^2 + 2}$

12) True

13) False

14)

15) $\theta = 25.7°$

Graphing Trigonometric Functions and Inverse Trigonometric Functions

For textbook reference you can use the free opnestax Precalculus text: https://openstax.org/details/books/precalculus
Chapter 6

Graph the function.

1) $y = \sin\left(x + \frac{\pi}{4}\right)$

2) $y = 2\sin\left(x - \frac{\pi}{4}\right)$

3) $y = \frac{2}{3}\cos\left(x + \frac{\pi}{3}\right)$

4) $y = 3 + \sin(2x-\pi)$

5) $y = \frac{1}{2} + \cos\left(2x - \frac{2\pi}{3}\right)$

6) $y = \frac{1}{3}\tan 2x$

7) $y = \frac{2}{3}\tan\left(\frac{1}{3}x - \frac{\pi}{3}\right)$

8) $y = \frac{1}{2}\cot\left(\frac{1}{2}x + \frac{\pi}{5}\right)$

9) $y = \csc\left(\frac{2}{5}x - \frac{\pi}{2}\right)$

10) $y = 2 + 4\sec\left(x + \frac{\pi}{5}\right)$

11) $y = \sin^{-1} x$

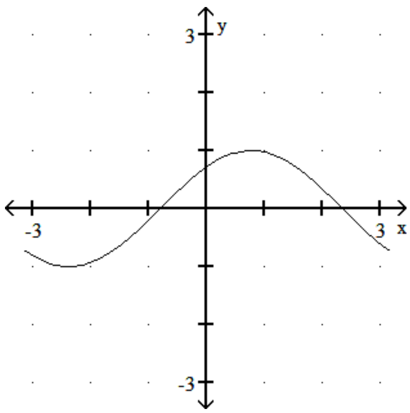
12) A generator produces an alternating current according to the equation $I = 80\sin 106\pi t$, where t is time in seconds and I is the current in amperes. What is the smallest time t such that $I = 40$?

13) **Find the exact value of the real number y.** $y = \arcsin\left(\frac{\sqrt{3}}{2}\right)$

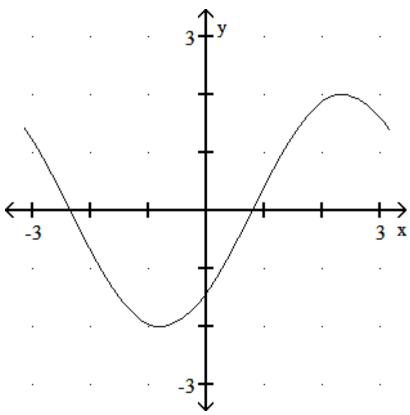
14) **Find the exact value of the real number y.** $\cos\left(\arcsin\frac{1}{4}\right)$

15) **Write the following as an algebraic expression in u, u > 0.** $\tan\left(\cos^{-1}\frac{u}{3}\right)$

1)


3
y
-3
3
x
-3

2)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y
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x
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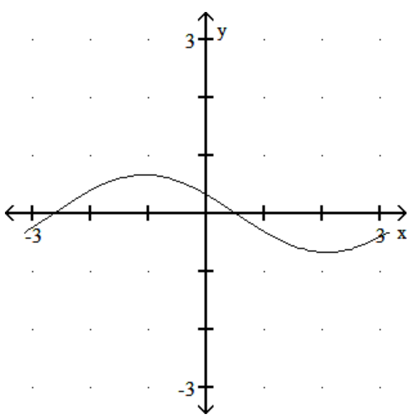
3)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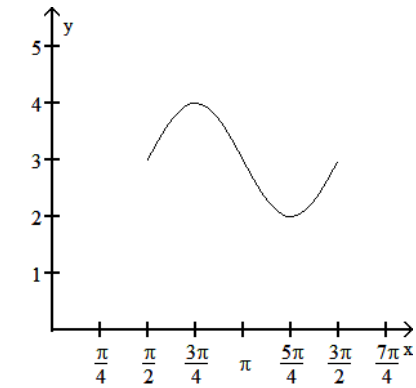
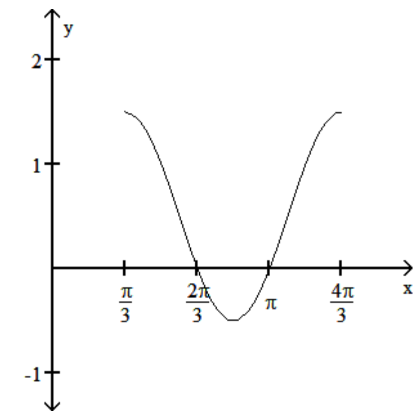
y
-3
3
x
-3

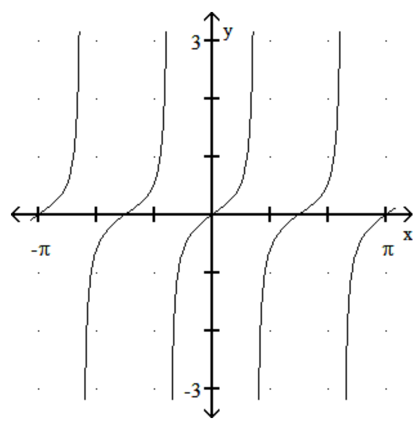
4)


y
5
4
3
2
1
π/4
π/2
3π/4
π
5π/4
3π/2
7π/4
x

5)

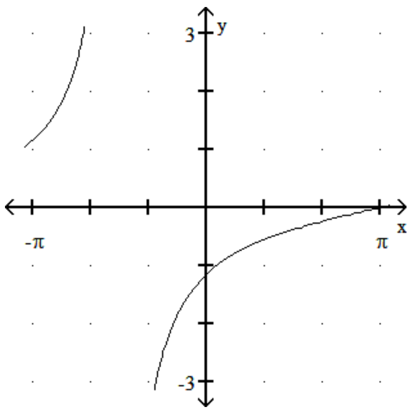

y
2
1
-1
π/3
2π/3
π
4π/3
x

6)

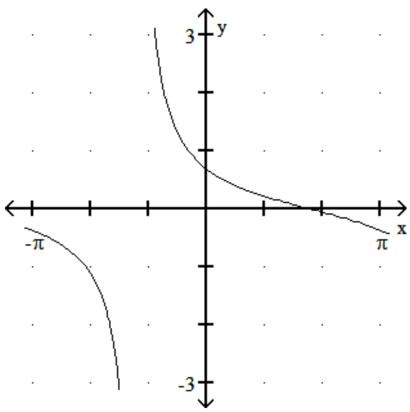

3
y
-π
π
x
-3

Answer Key

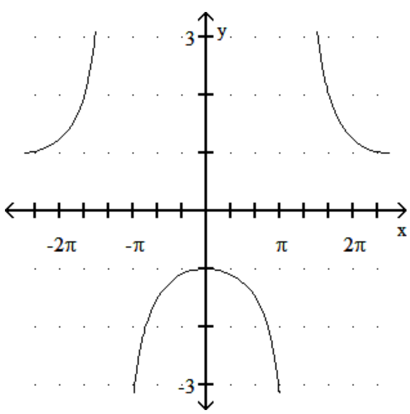
7)



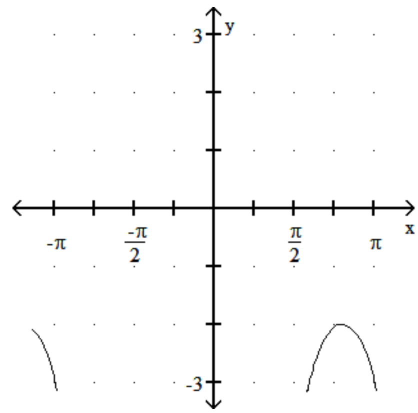
8)



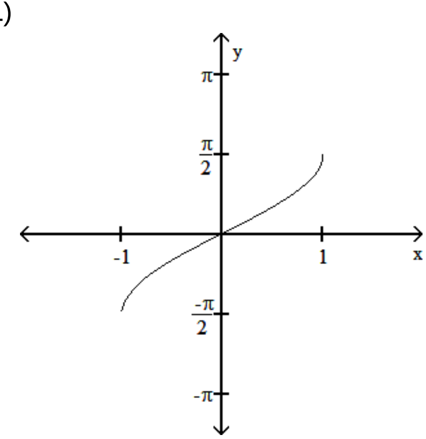
9)



10)



11)



12) $\frac{1}{636}$ sec

13) $\frac{\pi}{3}$

14) $\frac{\sqrt{15}}{4}$

15) $\frac{\sqrt{9 - u^2}}{u}$

Trigonometric Identities

For textbook reference you can use the free opnestax Precalculus text: https://openstax.org/details/books/precalculus
Sections 7.1 and 7.2

Use the fundamental identities to simplify the expression.

1) $\frac{1}{\cot^2\theta} + \sec\theta\cos\theta$ 1) ______________

2) $\sin^2\theta + \tan^2\theta + \cos^2\theta$ 2) ______________

3) $\cos x(\csc x - \sec x) - \cot x$ 3) ______________

Verify that each equation is an identity.

4) $\tan x(\csc x - \sin x) = \cos x$ 4) ______________

5) $\frac{\sec\theta - 1}{\tan\theta} = \frac{\tan\theta}{\sec\theta + 1}$ 5) ______________

6) $(\sec\alpha - \tan\alpha)(\sec\alpha + \tan\alpha) = 1$ 6) ______________

7) $\csc s - \sin s = \cos s \cot s$ 7) ______________

8) $(\sec\alpha + \tan\alpha)^2 = \frac{1 + \sin\alpha}{1 - \sin\alpha}$ 8) ______________

Use Identities to find the exact value.

9) $\cos 165°$ 9) ______________

10) $\cos\left(\frac{\pi}{12}\right)$ 10) ______________

Find the exact value of the expression using the provided information.

11) Find $\cos(s - t)$ given that $\cos s = -\frac{1}{2}$, with s in quadrant III, and $\cos t = -\frac{3}{5}$, with t in quadrant III. 11) ______________

Find the exact value by using a sum or difference identity.

12) $\tan 75°$ 12) ______________

13) $\sin 15°$ 13) ______________

Use a sum or difference identity to find the exact value.

14) $\sin\frac{7\pi}{24}\cos\frac{\pi}{8} - \cos\frac{7\pi}{24}\sin\frac{\pi}{8}$ 14) ______________

Answer Key
Testname: TRIG IDENTITIES

1) $\sec^2\theta$

2) $\sec^2\theta$

3) -1

4) $\tan x(\csc x - \sin x) = \tan x \cdot \csc x - \tan x \cdot \sin x = \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\sin x} - \dfrac{\sin x}{\cos x} \cdot \sin x = \dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x} = \dfrac{1 - \sin^2 x}{\cos x} = \dfrac{\cos^2 x}{\cos x} =$
$\cos x$

5) $\dfrac{\sec\theta - 1}{\tan\theta} = \dfrac{\sec\theta - 1}{\tan\theta} \cdot \dfrac{\sec\theta + 1}{\sec\theta + 1} = \dfrac{\sec^2\theta - 1}{\tan\theta(\sec\theta + 1)} = \dfrac{\tan^2\theta}{\tan\theta(\sec\theta + 1)} = \dfrac{\tan\theta}{\sec\theta + 1}$

6) $(\sec\alpha - \tan\alpha)(\sec\alpha + \tan\alpha) = \sec^2\alpha - \tan^2\alpha = 1$

7) $\csc s - \sin s = \dfrac{1}{\sin s} - \sin s = \dfrac{1 - \sin^2 s}{\sin s} = \dfrac{\cos^2 s}{\sin s} = \cos s \cdot \dfrac{\cos s}{\sin s} = \cos s \cot s$

8) $(\sec\alpha + \tan\alpha)^2 = \sec^2\alpha + 2\sec\alpha\tan\alpha + \tan^2\alpha = \dfrac{1}{\cos^2\alpha} + \dfrac{2\sin\alpha}{\cos^2\alpha} + \dfrac{\sin^2\alpha}{\cos^2\alpha} = \dfrac{1 + 2\sin\alpha + \sin^2\alpha}{\cos^2\alpha} = \dfrac{(1 + \sin\alpha)^2}{1 - \sin^2\alpha} =$
$\dfrac{(1 + \sin\alpha)^2}{(1 - \sin\alpha)(1 + \sin\alpha)} = \dfrac{1 + \sin\alpha}{1 - \sin\alpha}$

9) $\dfrac{-\sqrt{6} - \sqrt{2}}{4}$

10) $\dfrac{\sqrt{6} + \sqrt{2}}{4}$

11) $\dfrac{3 + 4\sqrt{3}}{10}$

12) $\sqrt{3} + 2$

13) $\dfrac{\sqrt{6} - \sqrt{2}}{4}$

14) $\dfrac{1}{2}$

Trigonometric Equations

For textbook reference and videos you can use section 7.5 of the free openstax Precalculus text:
https://openstax.org/details/books/precalculus

To access the section and videos,you can use use:
https://cnx.org/contents/_VPq4foj@11.14:aeVxcRIM@12/7-5-Solving-Trigonometric-Equations

Solve the equation for the interval $[0, 2\pi)$.

1) $\cos^2 x + 2 \cos x + 1 = 0$

2) $2 \sin^2 x = \sin x$

3) $\cos x = \sin x$

4) $\sin^2 x - \cos^2 x = 0$

Determine the solution set of each equation in radians (for x) or degrees (for θ) to the nearest tenth as appropriate.

5) $2 \sin^2 x + \sin x = 1$

6) $\dfrac{4 \tan \theta}{5 - \tan^2 \theta} = 1$

Solve the equation for solutions in the interval $[0, 2\pi)$.

7) $\sin 4x = \dfrac{\sqrt{3}}{2}$

8) $\sin x \cos x = \dfrac{1}{2}$

9) $\sin 2x + \sin x = 0$

Determine the solution set of each equation in radians (for x) or degrees (for θ) to the nearest tenth as appropriate.

10) $3 \cos^2 \theta + 2 \cos \theta = 1$

Answer Key:

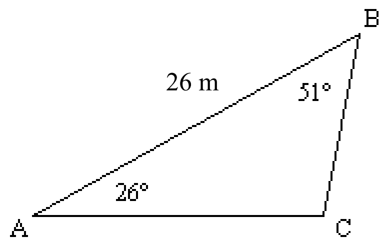
1) $\{\pi\}$
2) $\left\{0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$
3) $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$
4) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$
5) $\left\{\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{3\pi}{2} + 2n\pi\right\}$
6) $\{45° + 180°n, 101.3° + 180°n\}$
7) $\left\{\frac{\pi}{12}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{12}, \frac{7\pi}{6}, \frac{13\pi}{12}, \frac{5\pi}{3}, \frac{19\pi}{12}\right\}$
8) $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$
9) $\left\{0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$
10) $\{70.5° + 360°n, 180° + 360°n, 289.5° + 360°n\}$

Law of Sines and Law of Cosines

For reference, you can use chapters 8.1 and 8.2 of the openstax Precalculus text: **https://openstax.org/details/books/precalculus**

Solve the triangle.

1)



Solve the problem.

2) To find the distance AB across a river, a distance BC = 1053 m is laid off on one side of the river. It is found that B = 101.3° and C = 17.0°. Find AB rounded to the nearest meter.

Find the area of triangle ABC with the given parts. Round to the nearest whole number.

3) A = 26.4°
b = 12.3 in.
c = 7.7 in.

Find the missing parts of the triangle.

4) C = 35°30'
a = 18.76
c = 16.15

5) A = 79°
a = 32 yd
b = 65 yd

6) B = 63°30'
a = 12.20 ft
c = 7.80 ft

Find the missing parts of the triangle. (Find angles to the nearest hundredth of a degree.)

7) a = 27 ft
b = 32 ft
c = 41 ft

Find the area of triangle ABC with the given parts. Round to the nearest whole number.

8) a = 17.4 cm
b = 15.0 cm
c = 13.4 cm

9) Explain, in your own words, the situation called "the ambiguous case of the law of sines."

10) What happens if C = 90° when the law of cosines is applied in the form $C^2 = A^2 + B^2 - 2ab \cos C$?

Answer Key

1) C = 103°, a = 11.7 m, b = 20.7 m
2) 350 m
3) 21.1 $in.^2$
4) A_1 = 42°25', B_1 = 102°05', b_1 = 27.20;
A_2 = 137°35', B_2 = 6°55', b_2 = 3.35
5) no such triangle
6) b = 11.17 ft, A = 77°49', C = 38°41'
7) A = 41.14°, B = 51.24°, C = 87.62°
8) 97 cm^2
9) Answers will vary
10) Answers will vary

Vectors and Polar Coordinates

For reference, you can use chapters 8.1 and 8.2 of the openstax Precalculus text:
https://openstax.org/details/books/precalculus

Find the component form of the indicated vector.

1) Let $\mathbf{u} = \langle -9, -9 \rangle$, $\mathbf{v} = \langle -3, 3 \rangle$. Find $-\mathbf{u} + 9\mathbf{v}$.

Find the magnitude and direction angle (to the nearest tenth) for each vector. Give the measure of the direction angle as an angle in [0,360°].

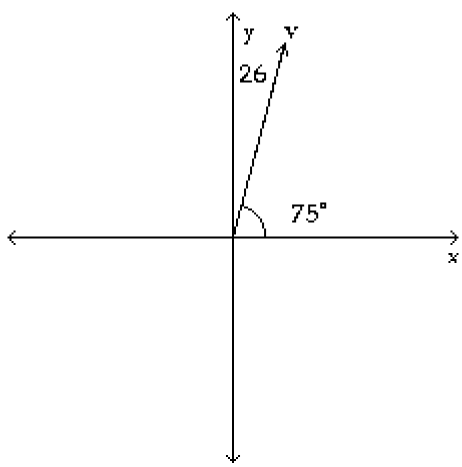
2) $\langle -3, -4 \rangle$

Vector v has the given magnitude and direction. Find the magnitude of the indicated component of v.

3) $\alpha = 38.3°$, $|\mathbf{v}| = 281$
Find the vertical component of **v**.

Write the vector in the form <a, b>.

4)



Two forces act at a point in the plane. The angle between the two forces is given. Find the magnitude of the resultant force.

5) forces of 25.0 and 31.8 lb, forming an angle of 162.8°

Find the dot product for the pair of vectors.

6) $\langle -16, 8 \rangle, \langle 0, 15 \rangle$

Find the angle between the pair of vectors to the nearest tenth of a degree.

7) $\langle 5, 5 \rangle, \langle -3, 8 \rangle$

8) Starting at point A, a ship sails 57 km on a bearing of 188°, then turns and sails 37 km on a bearing of 330°. Find the distance of the ship from point A.

9) Suppose you would like to cross a 209-foot wide river in a boat. Assume that the boat can travel 32 mph relative to the water and that the current is flowing west at the rate of 6 mph. If the bearing is chosen so that the boat will land at a point exactly across from its starting point, how long will it take for the boat to make the crossing? Give your answer to the nearest second.

The rectangular coordinates of a point are given. Express the point in polar coordinates with $r \geq 0$ and $0° \leq \theta < 360°$.

10) (2, -2)

Give the rectangular coordinates for the point.

11) (6, 225°)

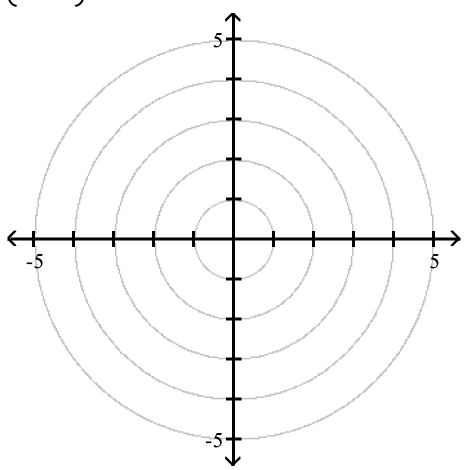
For the given rectangular equation, give its equivalent polar equation.

12) $2x + 3y = 6$

13) $x^2 + y^2 = 64$

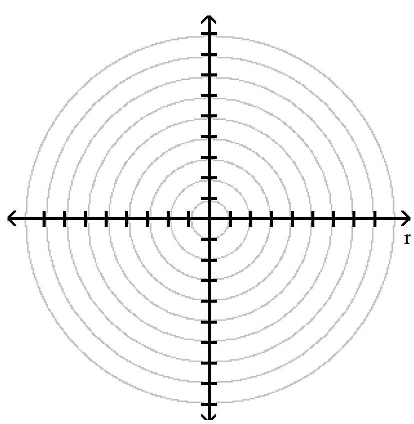
Plot the point.

14) $\left(4, \dfrac{9\pi}{4}\right)$



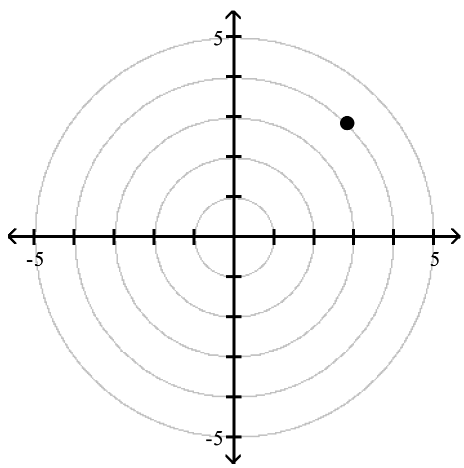
Graph the polar equation for θ in [0°, 360°).

15) $r = 4 + 4 \sin \theta$

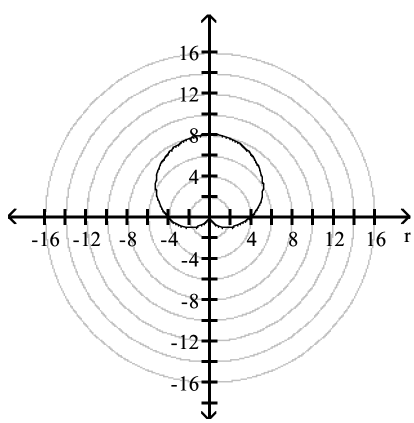


Answer Key

1) $\langle -18, 36 \rangle$
2) 5; 233.1°
3) 174.2
4) $\approx \langle 6.73, 25.11 \rangle$
5) 11 lb
6) 120
7) 65.6°
8) 36 km
9) 5 sec
10) $(2\sqrt{2}, 315°)$
11) $(-3\sqrt{2}, -3\sqrt{2})$
12) $r = \dfrac{6}{2 \cos \theta + 3 \sin \theta}$
13) r = 8
14)



15)



Polar Form of Complex Numbers

For reference, you can use chapters 8.5 of the openstax Precalculus text:
https://openstax.org/details/books/precalculus

1) **Graph the complex number.** $-5 - 4i$

Find the following quotient, and write the quotient in rectangular form, using exact values.

2) $\dfrac{12\text{cis } 158°}{3\text{cis } 38°}$

3) **Graph the complex number.** $-4i$

4) **Write the complex number in rectangular form.** $8\left(\cos \dfrac{\pi}{6} + i \sin \dfrac{\pi}{6}\right)$

5) **Write the complex number in rectangular form.** $9(\cos 180° + i \sin 180°)$

Write the complex number in trigonometric form r(cos θ + i sin θ), with θ in the interval [0°, 360°).

6) $5\sqrt{3} + 5i$

Find the product. Write the product in rectangular form, using exact values.

7) $[8 \text{ cis } 300°]\ [6 \text{ cis } 330°]$

8) **Find the given power. Write answer in rectangular form:** $(2 - 2i)^5$

9) **Find the given power. Write answer in rectangular form.:** $(-\sqrt{3} + i)^6$

10) **Find all cube roots of the complex number. Leave answers in trigonometric form:** $-125i$

11) **Find all cube roots of the complex number. Leave answers in trigonometric form:** $3 + 3i\sqrt{3}$

12) **Find all solutions of the equation. Leave answers in trigonometric form.:** $x^3 - 8 = 0$

13) **Find all solutions of the equation. Leave answers in trigonometric form.:** $x^5 - 32 = 0$

Use a table of values to graph the plane curve defined by the following parametric equations. Find a rectangular equation for the curve.

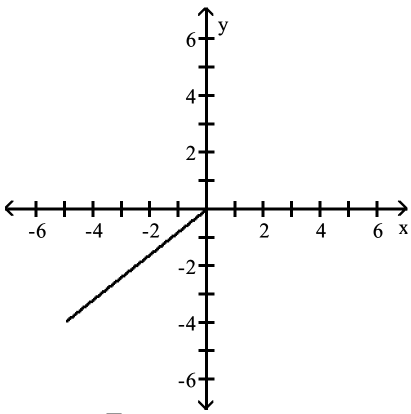
14) $x = 2t$, $y = t + 1$, for t in $[-2, 3]$

15) **Find a rectangular equation for the plane curve defined by the parametric equations:** $x = \sin t$, $y = 3 \cos t$

16) **Find a rectangular equation for the plane curve defined by the parametric equations.:** $x = \sec t$, $y = \tan t$

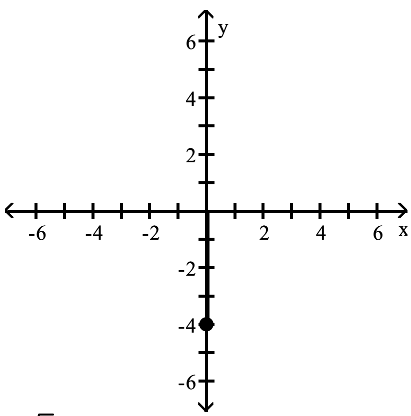
Answer Key

1)



2) $-2 + 2i\sqrt{3}$

3)



4) $4\sqrt{3} + 4i$

5) -9

6) $10(\cos 30° + i \sin 30°)$

7) $-48i$

8) $-128 + 128i$

9) -64

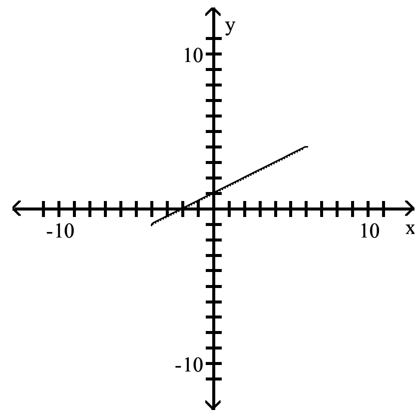
10) 5 cis 90°, 5 cis °210, 5 cis 330°

11) $\sqrt[3]{6}$ cis 20°, $\sqrt[3]{6}$ cis 140°, $\sqrt[3]{6}$ cis 260°

12) {2, 2 cis 120°, 2 cis 240°}

13) $\left\{2 \text{ cis } 0, 2 \text{ cis } \frac{2\pi}{5}, 2 \text{ cis } \frac{4\pi}{5}, 2 \text{ cis } \frac{6\pi}{5}, 2 \text{ cis } \frac{8\pi}{5}\right\}$

14)



$y = \frac{1}{2}x + 1$, for x in $[-4, 6]$

15) $9x^2 + y^2 = 9$

16) $x^2 - y^2 = 1$

Test Reviews

Trigonometric Identities and Equations Review

You can use chapter 7 in the free Precalculus text at openstax for reference: https://openstax.org/details/books/precalculus

Use the fundamental identities to simplify the expression.

1) $\dfrac{\cos^2\theta}{\sin^2\theta} + \csc\theta\sin\theta$

2) $\sin^2\theta + \tan^2\theta + \cos^2\theta$

Verify that each equation is an identity.

3) $\cot^2 x = (\csc x - 1)(\csc x + 1)$

4) $\sec\beta + \tan\beta = \dfrac{\cos\beta}{1 - \sin\beta}$

5) $(\sec\alpha - \tan\alpha)(\sec\alpha + \tan\alpha) = 1$

6) $\csc s - \sin s = \cos s \cot s$

Use Identities to find the exact value.

7) $\cos 255°$

Find the exact value by using a sum or difference identity.

8) $\tan 105°$

Find the exact value of the expression using the provided information.

9) Find $\tan(s + t)$ given that $\sin s = \dfrac{1}{4}$, with s in quadrant II, and $\sin t = -\dfrac{1}{2}$, with t in quadrant IV.

Find the exact value by using a sum or difference identity.

10) $\sin 15°$

Use trigonometric identities to find the exact value.

11) $\sin 25° \cos 35° + \cos 25° \sin 35°$

Use identities to find the indicated value for each angle measure.

12) $\cos 2\theta = \dfrac{2}{3}$ and θ terminates in quadrant I   Find $\sin\theta$.

13) $\cos 2\theta = \dfrac{1}{4}$ and θ terminates in quadrant III   Find $\cos\theta$.

Express the function as a trigonometric function of x.

14) $\cos 4x$

Verify that each equation is an identity.

15) $\cos(4u) = 2\cos^2(2u) - 1$

Write the product as a sum or difference of trigonometric functions.

16) $8 \cos 14° \cos 7°$

Find the exact value by using a half-angle identity.

17) $\sin 75°$

Determine all solutions of the equation in radians.

18) Find $\sin \frac{x}{2}$, given that $\sin x = \frac{1}{4}$ and x terminates in $0 < x < \frac{\pi}{2}$.

Solve the equation for the interval [0, 2π).

19) $\cos^2 x + 2 \cos x + 1 = 0$

20) $2 \sin^2 x = \sin x$

21) $\cos x = \sin x$

22) $\sin^2 x - \cos^2 x = 0$

Determine the solution set of each equation in radians (for x) or degrees (for θ) to the nearest tenth as appropriate.

23) $2 \sin^2 x + \sin x = 1$

Solve the equation for solutions in the interval [0, 2π).

24) $\sin 4x = \frac{\sqrt{3}}{2}$

25) $\sin x \cos x = \frac{1}{2}$

26) $\sin 2x + \sin x = 0$

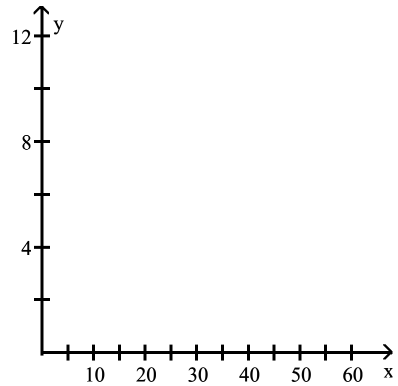
1) $\csc^2\theta$
2) $\sec^2\theta$
3) $\cot^2 x = \csc^2 x - 1 = (\csc x - 1)(\csc x + 1)$.
4) $\sec \beta + \tan \beta = \dfrac{1}{\cos \beta} + \dfrac{\sin \beta}{\cos \beta} = \dfrac{1 + \sin \beta}{\cos \beta} = \dfrac{1 + \sin \beta}{\cos \beta} \cdot \dfrac{1 - \sin \beta}{1 - \sin \beta} = \dfrac{1 - \sin^2 \beta}{\cos \beta(1 - \sin \beta)} = \dfrac{\cos^2 \beta}{\cos \beta(1 - \sin \beta)} = \dfrac{\cos \beta}{1 - \sin \beta}$
5) $(\sec \alpha - \tan \alpha)(\sec \alpha + \tan \alpha) = \sec^2\alpha - \tan^2\alpha = 1$
6) $\csc s - \sin s = \dfrac{1}{\sin s} - \sin s = \dfrac{1 - \sin^2 s}{\sin s} = \dfrac{\cos^2 s}{\sin s} = \cos s \cdot \dfrac{\cos s}{\sin s} = \cos s \cot s$
7) $\dfrac{\sqrt{2} - \sqrt{6}}{4}$
8) $-2 - \sqrt{3}$
9) $\dfrac{4\sqrt{3} + \sqrt{15}}{-11}$
10) $\dfrac{\sqrt{6} - \sqrt{2}}{4}$
11) $\dfrac{\sqrt{3}}{2}$
12) $\sin \theta = \dfrac{\sqrt{6}}{6}$
13) $\cos \theta = -\dfrac{\sqrt{101}}{4}$
14) $\cos^4 x - 6 \sin^2 x \cos^2 x + \sin^4 x$
15) $\cos(4u) = \cos[2(2u)] = 2 \cos^2(2u) - 1$
16) $4(\cos 21° + \cos 7°)$
17) $\dfrac{1}{2}\sqrt{2 + \sqrt{3}}$
18) $\dfrac{\sqrt{8 - 2\sqrt{15}}}{4}$
19) $\{\pi\}$
20) $\left\{0, \pi, \dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}$
21) $\left\{\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right\}$
22) $\left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right\}$
23) $\left\{\dfrac{\pi}{6} + 2n\pi, \dfrac{5\pi}{6} + 2n\pi, \dfrac{3\pi}{2} + 2n\pi\right\}$
24) $\left\{\dfrac{\pi}{12}, \dfrac{\pi}{6}, \dfrac{2\pi}{3}, \dfrac{7\pi}{12}, \dfrac{7\pi}{6}, \dfrac{13\pi}{12}, \dfrac{5\pi}{3}, \dfrac{19\pi}{12}\right\}$
25) $\left\{\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right\}$
26) $\left\{0, \dfrac{2\pi}{3}, \pi, \dfrac{4\pi}{3}\right\}$

Polar Forms and Vectors Review

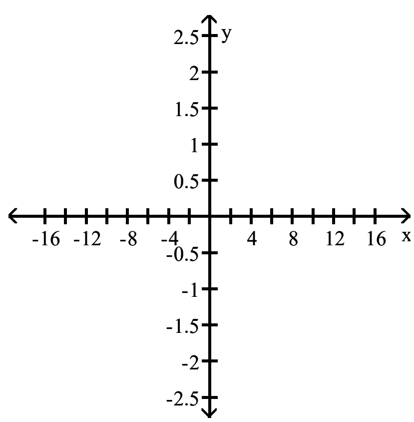
For reference, you can use chapters8 of the openstax Precalculus text: **https://openstax.org/details/books/precalculus**

1) $x = 5(t - \sin t)$, $y = 5(1 - \cos t)$, $0 \le t \le 4\pi$

Graph the cycloid for t in the indicated interval.



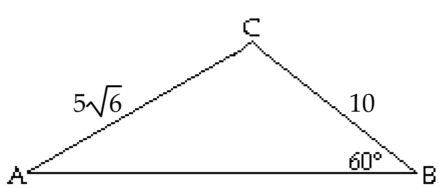
2) $x = t - \sin t$, $y = 1 - \cos t$, $-4\pi \le t \le 4\pi$



Find the missing parts of the triangle.

3) $A = 98°$
$b = 15.2$ ft
$a = 43.4$ ft

4)



5) $C = 35°30'$
$a = 18.76$
$c = 16.15$

Find the missing parts of the triangle. (Find angles to the nearest hundredth of a degree.)

6) $a = 27$ ft
$b = 32$ ft
$c = 41$ ft

Find the missing parts of the triangle.

7) $C = 112.5°$
$a = 5.30$ m
$b = 9.66$ m

Solve the problem.

8) Two ships leave a harbor together traveling on courses that have an angle of 125° between them. If they each travel 501 miles, how far apart are they (to the nearest mile)?

9) If $\mathbf{u} = \langle -5, 7\rangle$, $\mathbf{v} = \langle -7, 6\rangle$, and $\mathbf{w} = \langle -11, 2\rangle$, evaluate $\mathbf{u} \cdot (\mathbf{v} - \mathbf{w})$.

Find the angle between the pair of vectors to the nearest tenth of a degree.

10) $5\mathbf{i} - 3\mathbf{j}$, $5\mathbf{i} - 6\mathbf{j}$

Find the magnitude and direction angle (to the nearest tenth) for each vector. Give the measure of the direction angle as an angle in [0,360°].

11) $\langle -3, -4\rangle$

Two forces act at a point in the plane. The angle between the two forces is given. Find the magnitude of the resultant force.

12) forces of 52 and 54 newtons, forming an angle of 90°

Solve the problem.

13) A hot–air balloon is rising vertically 12 ft/sec while the wind is blowing horizontally at 5 ft/sec. Find the angle that the balloon makes with the horizontal.

14) A pilot wants to fly on a bearing of 65.3°. By flying due east, he finds that a 50–mph wind, blowing from the south, puts him on course. Find the ground speed of the plane.

15) A box weighing 80 lb is hanging from the end of a rope. The box is pulled sideways by a horizontal rope with a force of 24 lb. What angle, to the nearest degree, does the first rope make with the vertical?

Find the product. Write the answer in standard form.

16) $3i(3 + 6i)^2$

Simplify the power of i.

17) i^{79}

Find the following quotient, and write the quotient in rectangular form, using exact values.

18) $\dfrac{12\text{cis } 158°}{3\text{cis } 38°}$

Find the given power. Write answer in rectangular form.

19) $(1 + i)^{20}$

Find all specified roots.

20) Fifth roots of 1.

Find all solutions of the equation. Leave answers in trigonometric form.

21) $x^5 - 32 = 0$

For the given rectangular equation, give its equivalent polar equation.

22) $x - y = 10$

23) $x^2 + y^2 = 64$

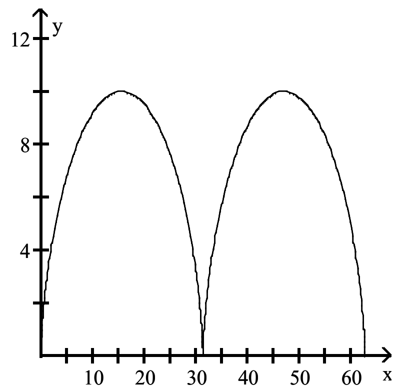
Find a rectangular equation for the plane curve defined by the parametric equations.

24) $x = t + 4$, $y = t^2$

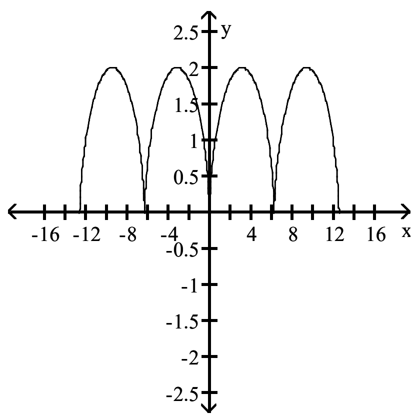
25) $x = 5 \tan t$, $y = 4 \cot t$

Answer Key

1)



2)



3) B = 20.3°, C = 61.7°, c = 38.6 ft

4) A = 45°, C = 75°, $c = 5\sqrt{3} + 5$

5) $A_1 = 42°25'$, $B_1 = 102°05'$, $b_1 = 27.20$;
$A_2 = 137°35'$, $B_2 = 6°55'$, $b_2 = 3.35$

6) A = 41.14°, B = 51.24°, C = 87.62°

7) c = 12.7 m, A = 22.7°, B = 44.8°

8) 889 mi

9) 8

10) 19.2°

11) 5; 233.1°

12) 75 newtons

13) 67.4°

14) 120 mph

15) 17°

16) −108 − 81i

17) −i

18) $-2 + 2i\sqrt{3}$

19) −1024

20) 1, $\text{cis}\,\frac{2\pi}{5}$, $\text{cis}\,\frac{4\pi}{5}$, $\text{cis}\,\frac{6\pi}{5}$, $\text{cis}\,\frac{8\pi}{5}$

21) $\left\{2\text{ cis }0,\ 2\text{ cis}\,\frac{2\pi}{5},\ 2\text{ cis}\,\frac{4\pi}{5},\ 2\text{ cis}\,\frac{6\pi}{5},\ 2\text{ cis}\,\frac{8\pi}{5}\right\}$

22) $r = \frac{10}{\cos\theta - \sin\theta}$

23) r = 8

24) $y = x^2 - 8x + 16$

25) $y = \frac{20}{x}$

Cumulative Review of Trigonometry

Final review

1) Find the supplement of an angle whose measure is 37°45′2″

2) 139°47′ + 108°48′

Convert the angle to decimal degrees and round to the nearest hundredth of a degree.

3) 45°31′46″

Convert the angle to degrees, minutes, and seconds.

4) 28.34°

Find the angle of smallest possible positive measure coterminal with the given angle.

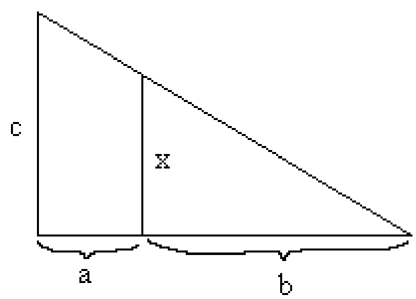
5) 840°

Suppose that θ is in standard position and the given point is on the terminal side of θ.

6) (6, 8); Find cos θ.

The triangles are similar. Find the missing side, angle or value of the variable.

7) x



a = 30 b = 90 c = 52

Evaluate the expression.

8) sin(-180°)

9) sec 270°

Identify the quadrant for the angle θ satisfying the following conditions.

10) sin $\theta > 0$ and cos $\theta < 0$

11) sec $\theta < 0$ and tan $\theta < 0$

Use the fundamental identities to find the value of the trigonometric function.

12) Find csc θ, given that sin $\theta = -\frac{2}{3}$ and θ is in quadrant IV.

Without using a calculator, give the exact trigonometric function value with rational denominator.

13) cos 60°

14) sec 45°

Suppose ABC is a right triangle with sides of lengths a, b, and c and right angle at C. Find the unknown side length .

15) Find csc A when $b = 8$ and $c = 17$

Find a solution for the equation. Assume that all angles are acute angles.

16) $\sin(2\beta + 15°) = \cos(3\beta - 25°)$

17) A fire is sighted due west of lookout A. The bearing of the fire from lookout B, 5.1 miles due south of A, is N 48° 22'W. How far is the fire from B (to the nearest tenth of a mile)?

Convert the degree measure to radians. Leave answer as a multiple of π.

18) 330°

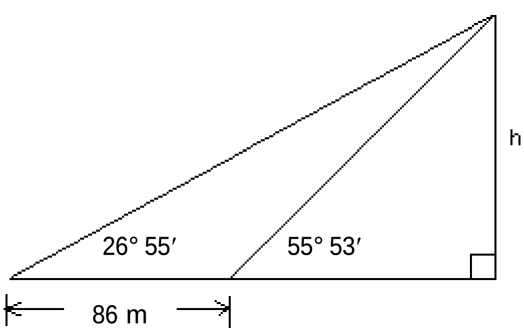
Convert the radian measure to degrees. Round to the nearest hundredth if necessary.

19) $-\frac{\pi}{5}$

Find the exact value without using a calculator.

20) $\sin \frac{3\pi}{4}$

21) Find h as indicated in the figure. Round your answer to the hundredths place.



Solve the problem.

22) Two wheels are rotating in such a way that the rotation of the smaller wheel causes the larger wheel to rotate. The radius of the smaller wheel is 6.5 centimeters and the radius of the larger wheel is 17.0 centimeters. Through how many degrees will the larger wheel rotate if the smaller one rotates 120°?

Graph the function over a one-period interval.

23) $y = \frac{1}{2}\cos 4\left(x - \frac{\pi}{3}\right)$

24) $y = \sin(2x - 180)$

Graph the function.

25) $y = \tan\left(\frac{1}{2}x - \frac{\pi}{6}\right)$

26) $y = 2 \csc(3x + 60)$

Solve the problem.

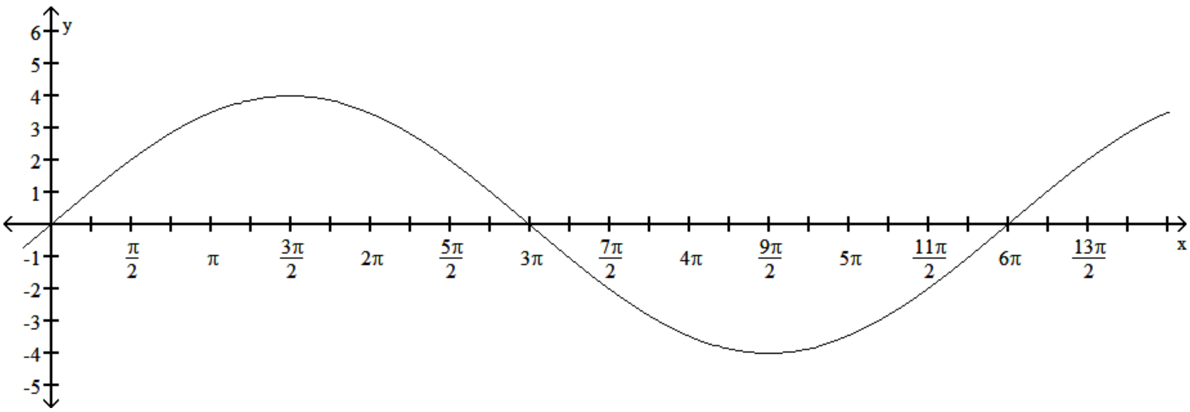
27) The temperature in Fairbanks is approximated by

$$T(x) = 37 \sin\left[\frac{2\pi}{365}(x - 101)\right] + 25,$$

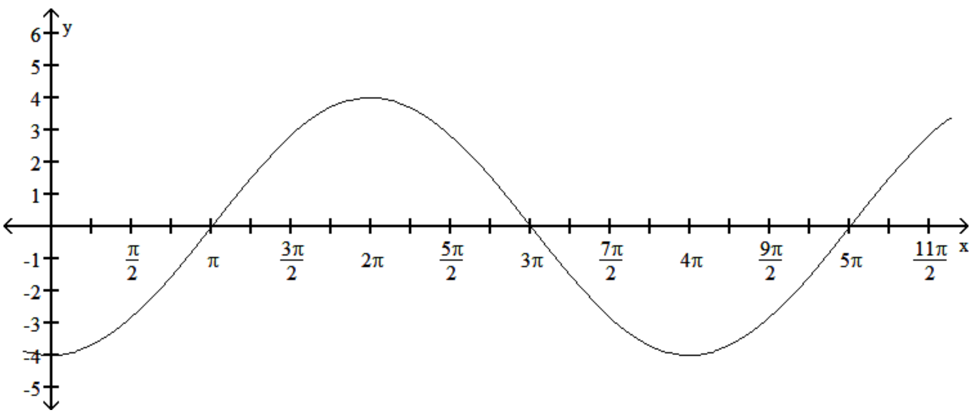
where $T(x)$ is the temperature on day x, with $x = 1$ corresponding to Jan. 1 and $x = 365$ corresponding to Dec. 31. Estimate the temperature on day 10.

The function graphed is of the form $y = a \sin bx$ or $y = a \cos bx$, where $b > 0$. Determine the equation of the graph.

28)



29)



Verify that each equation is an identity.

30) $\tan x(\csc x - \sin x) = \cos x$

31) $(\sec \alpha - \tan \alpha)(\sec \alpha + \tan \alpha) = 1$

32) $\sec \beta + \tan \beta = \dfrac{\cos \beta}{1 - \sin \beta}$

33) A wheel is rotating at 3 radians/sec, and the wheel has a 54-inch diameter. To the nearest foot, what is the speed of a point on the rim in ft/min?

Identify the quadrant for the angle θ satisfying the following conditions.

34) $\sec \theta < 0$ and $\tan \theta < 0$

35) Find $\cos(s + t)$ given that $\cos s = \frac{1}{3}$, with s in quadrant I, and $\sin t = -\frac{1}{2}$, with t in quadrant IV.

Use trigonometric identities to find the exact value.

36) $\sin 100° \cos 40° - \cos 100° \sin 40°$

Find the exact value by using a sum or difference identity.

37) $\sin 15°$

38) Find $\tan(s - t)$ given that $\sin s = -\frac{3\sqrt{13}}{13}$, with s in quadrant IV, and $\sin t = -\frac{\sqrt{10}}{10}$, with t in quadrant IV.

39) $\cos \theta = \frac{12}{13}$, $\sin \theta < 0$ Find $\sin(2\theta)$.

40) $\cos 2\theta = \frac{2}{3}$ and θ terminates in quadrant III Find $\cos \theta$.

Determine all solutions of the equation in radians.

41) Find $\cos \frac{\theta}{2}$, given that $\cos \theta = -\frac{3}{5}$ and θ terminates in $90° < \theta < 180°$.

Find the exact value

42) $\cos\left(\arcsin \frac{1}{4}\right)$

43) $\cos\left(2\arcsin \frac{3}{5}\right)$

Determine the solution set of each equation in radians (for x) or degrees (for θ) to the nearest tenth as appropriate.

44) $2 \sin^2 x + \sin x = 1$

Solve the equation for the interval $[0, 2\pi)$.

45) $\sin^2 x - \cos^2 x = 0$

Determine the solution set of each equation in radians (for x) or degrees (for θ) to the nearest tenth as appropriate.

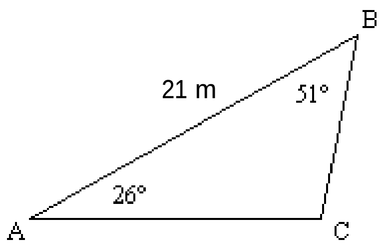
46) $3 \cos^2 \theta + 2 \cos \theta = 1$

Solve the equation for solutions in the interval [0°, 360°). Round to the nearest degree.

47) $\sin 2\theta = \cos \theta$

Solve the triangle.

48)



Find all th epossible values for the missing parts of the triangle.

49) $A = 23°$ $a = 35$ km $b = 55$ km

Find the missing parts of the triangle. (Find angles to the nearest hundredth of a degree.)

50) $a = 22$ ft $b = 32$ ft $c = 43$ ft

51) wo airplanes leave an airport at the same time, one going northwest (bearing 135°) at 407 mph and the other going east at 345 mph. How far apart are the planes after 3 hours (to the nearest mile)?

Find the magnitude and direction angle (to the nearest tenth) for each vector. Give the measure of the direction angle as an angle in [0,360°].

52) $\langle -5, 12 \rangle$

Vector v has the given magnitude and direction. Find the magnitude of the indicated component of v.

53) $\alpha = 25.9°$, $|v| = 85.6$ Find the horizontal component of v.

54) Two forces, of 45.2 and 17.0 lb, forming an angle of 141.9°, act at a point in the plane. Find the magnitude of the resultant force.

Find the quotient. Write the answer in standard form.

55) $\dfrac{9 - 3i}{5 - 7i}$

Write the complex number in trigonometric form $r(\cos \theta + i \sin \theta)$, with θ in the interval [0°, 360°).

56) $-6 - 6i\sqrt{3}$

Find the given power. Write answer in rectangular form.

57) $(1 + i)^{20}$

Find all solutions of the equation. Leave answers in trigonometric form.

58) $x^5 - 243 = 0$

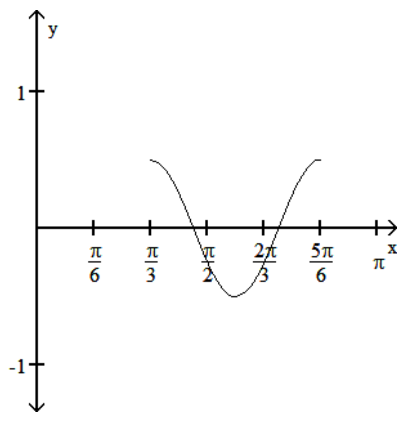
59) Find an equivalent equation in rectangular coordinates. $r = \cos \theta$

Find a rectangular equation for the plane curve defined by the parametric equations.

60) $x = t^2 + 1$, $y = t^2 - 1$

Answers:

1) $142°14'58''$
2) $248°35'$
3) $45.53°$
4) $28°20'24''$
5) $120°$
6) $\frac{3}{5}$
7) $x = 39$
8) 0
9) Undefined
10) Quadrant II
11) Quadrant II
12) $-\frac{3}{2}$
13) $\frac{1}{2}$
14) $\sqrt{2}$
15) $\frac{17}{15}$
16) $20°$
17) 7.7 mi
18) $\frac{11\pi}{6}$
19) $-36°$
20) $\frac{\sqrt{2}}{2}$
21) 66.55 m
22) $45.88°$
23)



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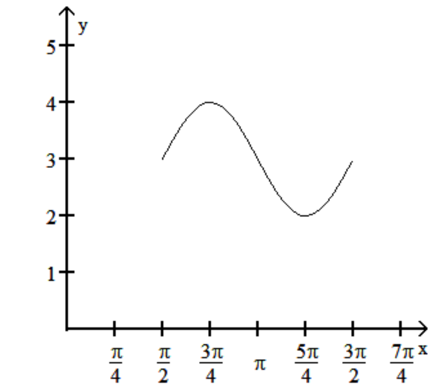

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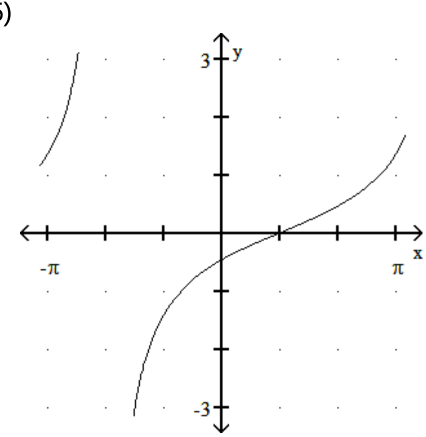

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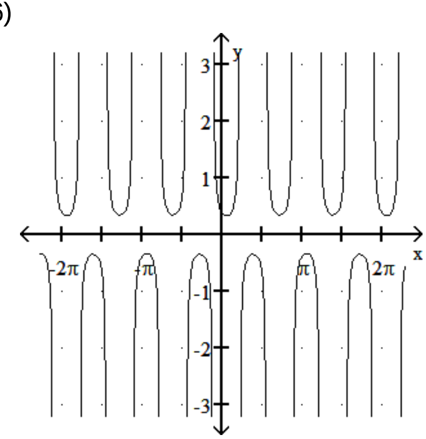

27) $-12°$
28) $y = 4 \sin\left(\frac{1}{3}x\right)$
29) $y = -4 \cos\left(\frac{1}{2}x\right)$

30) $\tan x(\csc x - \sin x) = \tan x \cdot \csc x - \tan x \cdot \sin x = \frac{\sin x}{\cos x}$

$\cdot \frac{1}{\sin x} - \frac{\sin x}{\cos x} \cdot \sin x = \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} = \frac{1 - \sin^2 x}{\cos x} =$

$\frac{\cos^2 x}{\cos x} = \cos x$

31) $(\sec \alpha - \tan \alpha)(\sec \alpha + \tan \alpha) = \sec^2\alpha - \tan^2\alpha = 1$

32) $\sec \beta + \tan \beta = \frac{1}{\cos \beta} + \frac{\sin \beta}{\cos \beta} = \frac{1 + \sin \beta}{\cos \beta} = \frac{1 + \sin \beta}{\cos \beta} \cdot$

$\frac{1 - \sin \beta}{1 - \sin \beta} = \frac{1 - \sin^2 \beta}{\cos \beta(1 - \sin \beta)} = \frac{\cos^2 \beta}{\cos \beta(1 - \sin \beta)} =$

$\frac{\cos \beta}{1 - \sin \beta}$

33) 405 ft/min

34) Quadrant II

35) $\frac{\sqrt{3} + 2\sqrt{2}}{6}$

36) $\frac{\sqrt{3}}{2}$

37) $\frac{\sqrt{6} - \sqrt{2}}{4}$

38) $-\frac{7}{9}$

39) $-\frac{120}{169}$

40) $\cos \theta = -\frac{\sqrt{30}}{6}$

41) $\frac{\sqrt{5}}{5}$

42) $\frac{\sqrt{15}}{4}$

43) $\frac{4\sqrt{3}-3}{10}$

44) $\left\{\frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi, \frac{3\pi}{2} + 2n\pi\right\}$

45) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

46) {70.5° + 360°n, 180° + 360°n, 289.5° + 360°n}

47) {30°, 90°, 150°, 270°}

48) C = 103°, a = 9.4 m, b = 16.7 m

49) $B_1 = 38°$, $C_1 = 119°$, $c_1 = 78$ km

$B_2 = 142°$, $C_2 = 15°$, $c_2 = 23$ km

50) A = 29.76°, B = 46.22°, C = 104.02°

51) 2085 mi

52) 13; 112.6°

53) 77.0

54) 33.5 lb

55) $\frac{33}{37} + \frac{24}{37}i$

56) 12(cos 240° + i sin 240°)

57) -1024

58) $\left\{3 \text{ cis } 0, 3 \text{ cis } \frac{2\pi}{5}, 3 \text{ cis } \frac{4\pi}{5}, 3 \text{ cis } \frac{6\pi}{5}, 3 \text{ cis } \frac{8\pi}{5}\right\}$

59) $x^2 + y^2 = x$

60) $y = x - 2, x \geq 1$

Articles

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WHERE DISCOVERIES BEGIN

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  - *Mathematics for the Life Sciences* by Erin N. Bodine, Suzanne Lenhart & Louis J. Gross
  - The University of Arizona's Biology Project website: http://www.biology.arizona.edu/
  - The Discovering Radians Activity comes from: http://mrsnicoleburns.weebly.com/uploads/8/6/7/0/8670276/_1__discovering_radians_activity__page_1_.pdf

- Content was developed for Los Angeles Mission College Trigonometry classes which use the Pearson textbook *Trigonometry* 11th Edition, by Lial/Hornsby/Schneider/Daniels, and some of the content was derived from Pearson's TestGen testbanks for this textbook.

- The Open Educational Resource Textbook from OpenStax *Precalculus* by Abramson is referenced for students as a resource for content review.

- The article is from the Journal of Experimental Biology