COURSE COMPETENCIES

1. **Solve systems of linear equations with two unknowns**
   Criteria
   - You solve a system of linear equations with two unknowns.
   - You apply mathematical solutions to applied technical problems as described by linear equations
   - You understand linear and non-linear graphs, both how to create and how to read

BACKGROUND

This module will show the students more applications using Kirchhoff's Voltage and Current Laws. Applying basic mathematic principles to analyze circuits that are more complicated. The students should be able to proficiently solve linear equations. The first Blackboard practice set is linear equations.

EXPLICIT CONNECTIONS

It is important that each student understands how to solve systems of equations. While each student needs to understand all methods presented, typically a student will prefer one method.

NOTES TO SELF

- Encourage each student to check their answers (first) and to help one another with theirs if needed. They often do not want to take the time to check their answers.

Duration Minutes	Lesson	Suggested Structure
15	Problem Set 5.1 – Gym Membership	Group
15	Problem Set 5.2 – Shapes	Group
20	Lecture – Systems of Equations	Cohort
15	Blackboard – Practice Set 1 - Linear Equation Review	Individual
15	Problem Set 5.3 – Application problem - Graphing method	Group
10	Blackboard – Practice Set 2 - Graphing Method	Individual
20	Lecture – Substitution Method	Cohort
15	Problem Situation 5.4 – Substitution Method	Group
15	Blackboard – Practice Set 3 - Substitution Method	Individual
20	Lecture – Addition Method	Cohort
15	Problem Situation 5.5 – Elimination Method (addition method)	Group
20	Lecture: Determinants	Cohort
15	Problem Situation 5.6 – Determinants Problem	Group
20	Lecture: Cramer's Rule	Cohort
20	Problem Situation 5.7 – Cramer's Rule Problem 1 & 2	Group
10	Lecture: Calculator (optional)	Cohort
15	Problem Situation 5.8 – Calculator Problem (optional)	Group
15	Blackboard – Practice Set 4 - Determinants	Individual
10	Blackboard – Practice Set 5 - Systems of Equations	Individual
20	Quiz	Cohort

Lesson	***Objectives***	***Material***
5.1	System of equations	Gym membership
5.2	System of equations	Shapes
5.3	Systems of Equations: Graphing method	Graphing Method
5.4	Systems of Equations: Substitution method	Substitution method
5.5	Systems of Equations: Elimination method	Elimination method
5.6	Systems of Equations: Cramer's Rule	Cramer's Rule

Prerequisite Assumptions
Before beginning the lesson, students should understand and be able to apply;

- Kirchhoff's Voltage Law
- Kirchhoff's Current Law
- Ohm's Law
- Power Rule

Specific Objectives
By the end of this lesson, you should understand;

- ✓ Graphing Linear Equations
- ✓ Solving a system of equations

By the end of this lesson, you should be able to;

- ✓ Use the graphing method to find the solution of a Linear System of Equations
- ✓ Use the substitution method to find the solution of a Linear System of Equations
- ✓ Use the elimination method to find the solution of a Linear System of Equations
- ✓ Use the Cramer's rule to find the solution of a Linear System of Equations
- ✓ Graph non-linear equations
- ✓ Use the quadratic equation to solve second order equations

Problem Situation 5.1 – Gym Membership

1) You find this flyer and think you want to start working out. Pick the number of months that you want to try working out. Predict which plan that you think will work for you, based solely on cost (because you just do not have a lot of money), using the number of months you chose.

I choose 3 months and the Base membership.

2) Write equations for the costs of each of the three scenarios based on any number of months you chose. *(the number of months is the independent variable)*

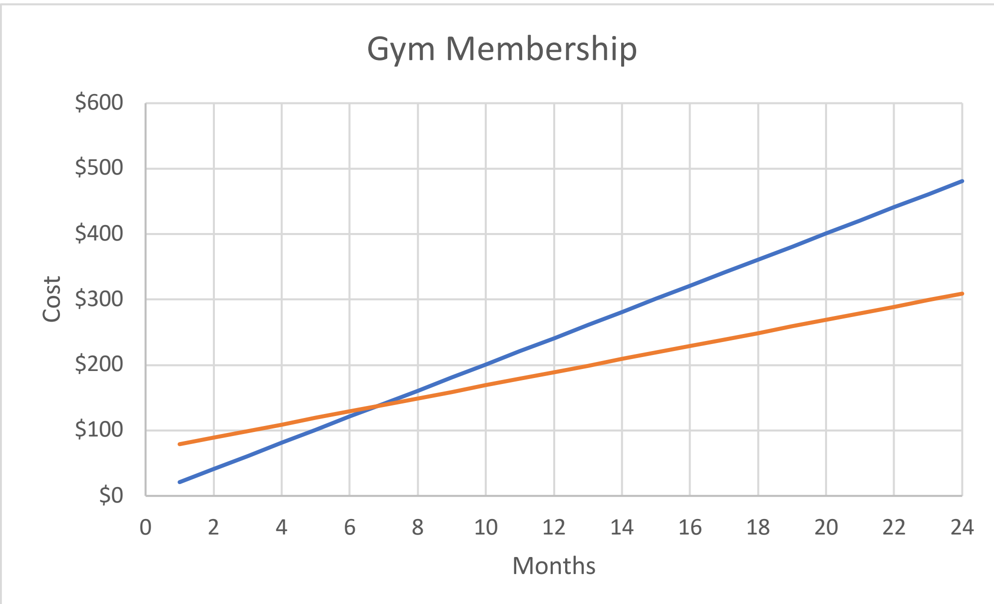
I need to find equations for each scenario.
I will set the variable C = total cost and m = number of months.
Scenario A: $C = \$1 + \$20 * m$
Scenario B: $C = \$69 + \$10 * m$
Scenario C: $0 < m < 12 \rightarrow C = \$199$ and $12 < m < 12 \rightarrow C = \$398$

3) Which two of these equations are linear equations?

Two of the scenarios (A and B) have equations that are linear functions with one dependent variable (on the number of months) and one independent variable. The other scenario, C, is a constant.

4) Graph each of the scenarios on the following graph. You might want to create a table.

Months	Cost ($) A	Cost ($) B	Cost ($) C
1	21	79	199
3	61	99	199
7	139	139	199
10	201	169	199
24	481	309	398



5) Is there a point in time in which the costs of the first two plans are the same? This is the **point of the intersection**, (*x*,*y*). This *ordered pair* of numbers is the *solution* for each of the equations.

To find the solution I need to find the intersection of the two linear equations.
By the graph, at about 7 months the costs of each plan intersect.

A **system of equations** is a set of equations with *two or more* variables that represent a *single* situation. The two variables, months and costs, are in both equations. The solution to the point of intersection is the number of months that result in the same cost.

- ✓ **One** unknown variable (solution) can be found with a *minimum* of **one** linear equation.
- ✓ **Two** unknown variables can be found with a *minimum* of **two** linear equations.
- ✓ **Three** unknown variables can be found with a *minimum* of **three** linear equations.
- ✓ Examples of a system of equations:

$x + y = 6$	$a + 2b = 4$
$x - 2y = 6$	$2a + b = 5$

How could you find the point of intersection for these two examples?

Problem Situation 5.2 – Shapes

a) Shapes Act 1: View this video Shapes. What is happening?

Every shape (triangle, square, and circle) has a 'value' assigned to it.
They are 'dumping' different numbers of shapes into 3 different buckets and then add up the total value of all the shapes in each bucket

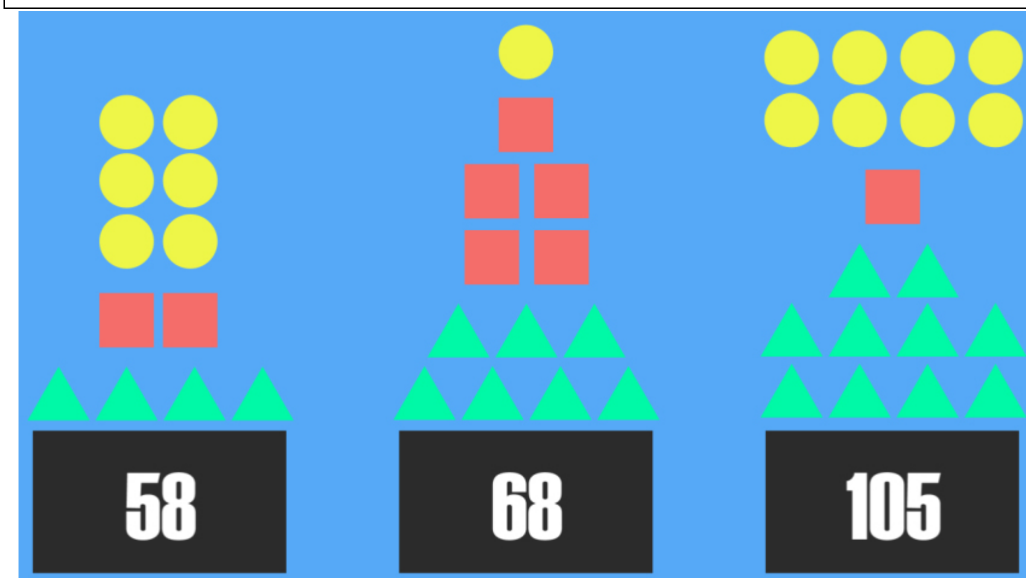
b) What would you guess each shape is worth?

On the average, each shape is worth about 5 points. It depends on the number of each shape. We have three different examples so we should be able to get three equations.

c) What information do you need to determine the points value assigned to each shape?

How many of each shape, the total value of each bucket; I will need to assign variables to each shape as unknowns.

Show Act 2 to confirm their ideas.



d) Determine the points assigned to each shape.

To determine the points I will first write down what I know starting with declaring the variables.

C = circles, S = squares, T = triangles

$6C + 2S + 4T = 58$

$C + 5S + 7T = 68$

$8C + S + 10T = 105$

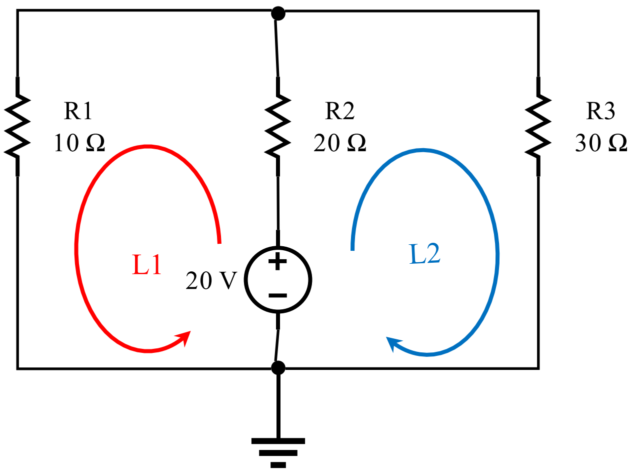
C = 4 circles, S = 3 squares, T = 7 triangles

The key to this is that the students derive the **three** equations. Most students will use trial and error to find the correct answer after they get the equations. I usually have each group go through their method aloud. Show Act 3.

<u>Problem Situation 5.3 – Application problem - Graphing method</u>

Mesh analysis uses Kirchhoff's Voltage Law and Ohm's Law. The results are typically a system of equations.

1) Graph the following linear equations to determine the solution for the currents. Make sure to label the graph well.



$L1\!: \ 0 = 20V - V_{R2} - V_{R1}$

$0 = 20V - (I_1 + I_2)R_2 - I_1 R_1$

$30I_1 + 20I_2 = 20V$

$3I_1 + 2I_2 = 2V$

$L2\!: \ 0 = 20V - V_{R2} - V_{R1}$

$0 = 20V - (I_1 + I_2)R_2 - I_2 R_3$

$20I_1 + 50I_2 = 20V$

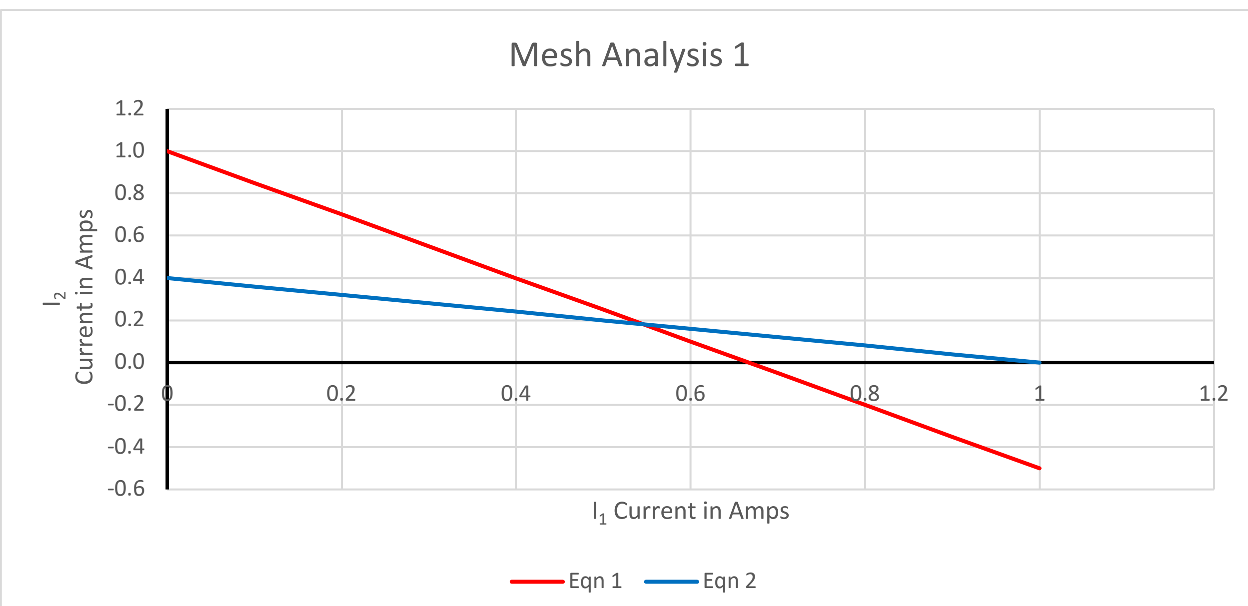
$2I_1 + 5I_2 = 2V$

$3I_1 + 2I_2 = 2$

I_1	I_2
0.00	1.00
0.10	0.85
0.20	0.70
0.30	0.55
0.40	0.40

$2I_1 + 5I_2 = 2$

I_1	I_2
0.00	0.40
0.10	0.36
0.20	0.32
0.30	0.28
0.40	0.24



2) The graphical solution is the ordered pair (x, y) at the intersection. Identify the intersection point..

The intersection of these two lines is at about (0.55, .19). This would be 500 mA for I_1 and 190 mA for I_2.

3) Verify that the solution is correct by applying Kirchhoff's Voltage & Current Laws to the circuit.

I need to verify that my solution is reasonably close to correct. Use KVL for L1 and L2.

$L1\text{: } 3I_1 + 2I_2 = 2V \rightarrow 3 * 0.55A + 2 * 0.19A = 2.03V \cong 2V$

$L2\text{: } 2I_1 + 5I_2 = 2V \rightarrow 2 * 0.55A + 5 * 0.19A = 2.05V \cong 2V$

4) Build your expertise and confidence by solving the following two systems of equations using the *graphical* method.

a)

x + *y* = 6

x – 2*y* = 6

b)

a + 2*b* = 4

2*a* + *b* = 5

I will determine at least two coordinates for each line. I can then connect the points to graph the lines. The intersection of the two lines is the solution to the systems of equations.

x + y = 6

x	y
0	6
6	0

x – 2y = 6

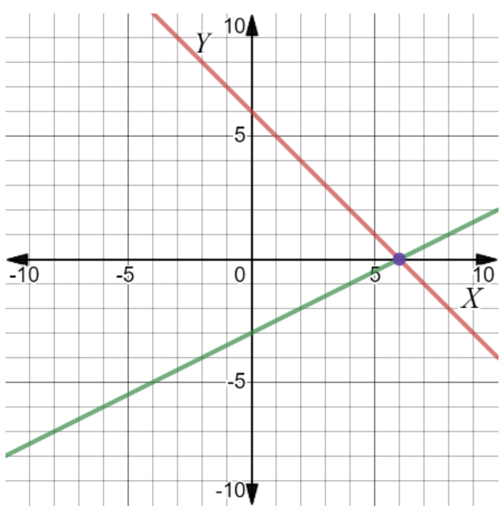
x	y
0	-3
6	0

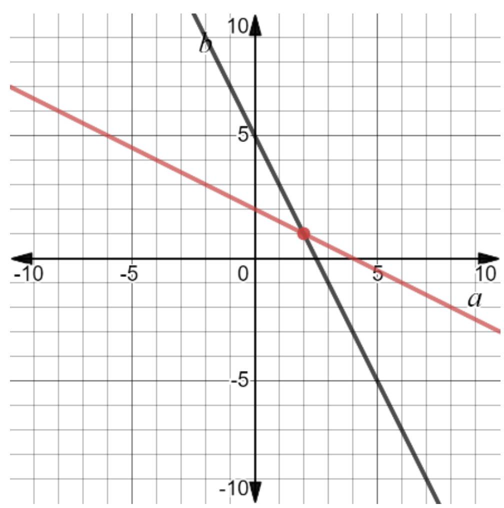
a +2b = 4

a	b
0	2
4	0

2a + b = 5

a	b
0	5
2.5	0





The solution is (6, 0). The solution is (2, 1).

Problem Situation 5.4 – Substitution Method

Another method for solving a linear system of equations is the substitution method.
To solve using the substitution method:

- "Solve" one of the equations by *isolating* one variable on one side of the equality in terms of the other variable. Often you will pick the variable that 'looks' like it would be easiest to isolate.
- Substitute that isolated variable into the other equation and then solve for the **remaining** variable.
- Use the value that you just found and substitute it into one of the original equations and solve for the original, isolated variable.

Example:

$Eqn\ 1: 4x - 5y = 6$
$Eqn\ 2: x + 2y = 8$

$$Eqn\ 1:\ 4x - 5y = 6 \quad \rightarrow \quad 4x = 6 + 5y \quad \rightarrow \quad x = \frac{6+5y}{4} \rightarrow \text{x is isolated in terms of y}$$

$$Eqn\ 2:\ x + 2y = 8 \quad \text{substitute for x:}\ \left(\frac{6+5y}{4}\right) + 2y = 8 \ \rightarrow\ (6+5y) + 4*2y = 4*8$$

$$6 + 5y + 8y = 32 \quad \rightarrow \quad 6 + 13y = 32 \quad \rightarrow \quad 13y = 32 - 6 \quad \rightarrow \quad y = \frac{26}{13} \quad \rightarrow \quad \boldsymbol{y = 2}$$

Substitute value for y back into original equation:

$$4x - 5*2 = 6 \quad \rightarrow \quad 4x - 10 = 6 \quad \rightarrow \quad 4x = 16 \quad \rightarrow x = \frac{16}{4} \quad \rightarrow \quad \boldsymbol{x = 4}$$

Check the solution: The ordered pair (4, 2) should make **both** equations true.

$Eqn\ 1: (4*4) - (5*2) = \mathbf{6}\ \surd$
$Eqn\ 2: 4 + (2*2) = \mathbf{8}\ \ \surd$

1) Use substitution to determine the solution for the following systems of equations.

a) $3x - 2y = 6$
$2x - 3y = 4$

I need to isolate one of the variables out of one of the equations.

I will isolate *x* in equation 1.

$Eqn\ 1: 3x - 2y = 6 \rightarrow x = \frac{6+2y}{3}$ Now I will substitute x into equation 2.

$$Eqn\ 2:\ 2x - 3y = 4 \rightarrow \quad 2\left(\frac{6+2y}{3}\right) - 3y = 4 \rightarrow \quad 4 + \frac{4}{3}y - 3y = 4 \rightarrow \quad \frac{-5}{3}y = 0 \rightarrow \quad \boldsymbol{y = 0}$$

Now I will replace y with its value and solve for x.

It does not matter which equation I use so I will use equation 1. $3x - 2*0 = 6 \rightarrow \boldsymbol{x = 2}$

I need to validate that I have the correct answers. $3*2 - 2*0 = 6\ \surd$ and $2*2 - 3*0 = 4\ \ \surd$

b) $5a + b = 15$

$a + 5b = 27$

I need to isolate one of the variables in one of the equations. I will isolate *b* in equation 1.

$Eqn\ 1: 5a\ + b = 15 \rightarrow b =\ 15 - 5a$. Now I will substitute x into equation 2

$Eqn\ 2:\ a + 5b = 27 \rightarrow a + 5(15 - 5a) = 27 \rightarrow a - 25a = 27 - 75 \rightarrow \boldsymbol{a = 2}$

Now I will replace a with its value and solve for b.

It does not matter which equation I use so I will use equation 1. $5 * 2\ + b = 15 \rightarrow \boldsymbol{b = 5}$

I need to validate that I have the correct answers. $5 * 2 + 5 = 15\ \ \surd$ and $2 + 5 * 5 = 27\ \surd$

c) $x\ +\ y\ =\ 6$
$x - 2y\ =\ 6$

I need to isolate one of the variables in one of the equations. I will isolate *x* in equation 1.

$Eqn1: x + y\ =\ 6\ \rightarrow x =\ 6 - y$ Now I will substitute x into equation 2.

$Eqn2:\ (6 - y) - 2y\ =\ 6 \rightarrow 6 - 3y = 6 \rightarrow \boldsymbol{y = 0}$

Now I will replace y with its value and solve for x.

It does not matter which equation I use so I will use equation 1. $x + 0\ =\ 6 \rightarrow \boldsymbol{x = 6}$

I need to validate that I have the correct answers. $6 + 0\ =\ 6\ \surd$ and $6 - 2 * 0 = 6\ \surd$

d) $a\ +\ 2b\ =\ 4$
$2a\ +\ b\ =\ 5$

I need to isolate one of the variables in one of the equations. I will isolate *a* in equation 1.

$Eqn1: a\ +\ 2b\ =\ 4 \rightarrow a =\ 4 - 2b$ Now I will substitute a into equation 2.

$Eqn2:\ 2(4 - 2b) +\ b\ =\ 5 \rightarrow 8 - 3b = 5 \rightarrow \boldsymbol{b = 1}$

Now I will replace b with its value and solve for a. I will use equation 1. $a\ +\ 2 * 1\ =\ 4 \rightarrow \boldsymbol{a = 2}$

I need to validate that I have the correct answers. $2\ +\ 2 * 1\ =\ 4\ \surd$ and $2 * 2\ +\ 1\ =\ 5\ \surd$

2) Using the substitution method, solve for the currents in our **initial** circuit.

$3I_1 + 2I_2 = 2$
$2I_1 + 5I_2 = 2$

I need to isolate one of the variables in one of the equations. I will isolate I_1 in equation 1.

$Eqn1: 3I_1 + 2I_2 = 2\ \rightarrow I_1 = \frac{2-2I_2}{3}$ Now I will substitute I_1 into equation 2.

$Eqn2:\ 2\left(\frac{2 - 2I_2}{3}\right)_1 + 5I_2 = 2 \rightarrow 4 - 4I_2 + 15I_2 = 6 \rightarrow 11I_2 = 2 \rightarrow \boldsymbol{I_2 = 182mA}$

Now I will replace I_2 with its value and solve for I_1. I will use equation 1.

$3I_1 + 2 * 182mA = 2\ \rightarrow \boldsymbol{I_1 = 545mA}$

Validate answers are correct: $3 * 545mA + 2 * 182mA = 2\ \surd$ and $2 * 545mA + 5 * 182mA = 2\ \surd$

Problem Situation 5.5 – Elimination Method (addition method)

Another method for solving a linear system of equations with two equations and two unknowns is called the **Elimination Method**. To solve using the elimination method;

- Set up the equations so that when you add them together one of the variables is eliminated.
- Determine the value of the remaining variable
- Insert the solution for the solved variable into one of the equations to find the other variable.

EXAMPLE:

$Eqn\ 1:\ 4x - 5y = 6$
$Eqn\ 2:\ x + 2y = 8$

- "Change" Equation 2

$Eqn\ 2:\ x + 2y = 8 \quad \rightarrow (mult\ by - 4) \rightarrow\ (-4 * x) + (-4 * 2y) = (-4 * 8) \quad \rightarrow\ \mathbf{-4x - 8x = -32}$

- Add equations to eliminate "*x*"

$Eqn\ 1:\quad 4x - 5y = 6$
$\underline{Eqn\ 2:\ -4x - 8x = -32}$

$$0x - 13y = -26$$

- And solve for "*y*"

$y = \frac{-26}{-13} \quad \rightarrow \quad \boldsymbol{y = 2}$

- Substitute "*y*" back into original equation and solve for "*x*"

$4x - 5 * 2 = 6 \ \rightarrow\ 4x - 10 = 6 \ \rightarrow\ 4x = 16 \ \rightarrow x = \frac{16}{4} \ \rightarrow\ \boldsymbol{x = 4}$

- Check the solution:

$(4 * 4) - (5 * 2) = 6\ \surd$
$4 + (2 * 2) = 8\ \surd$

1) Using the elimination method, determine the solution for the following of systems of equations. Check your answers.
   a)
   $2a - 4 = b$
   $a + 2b = 7$

To use the elimination method, I have to line up the variables of the equations in standard form :
$2a - b = 4$ Multiply the top equation by 2→ $4a - 2b = 8$
$a + 2b = 7$ $\qquad \underline{\ \ a + 2b = 7}$
$5a \qquad = 15 \ \rightarrow\ \boldsymbol{a = 3}$
Now I can substitute the value of "*a*" into one of the equations to determine the value of "*b*".
$2 * 3 - b = 4 \rightarrow \boldsymbol{b = 2}$
I need to validate that I have the correct answers. $2 * 3 - 2 = 4\ \ \surd$ and $3 + 2 * 2 = 7\ \ \surd$

b)

$4x + 6 = y$
$3y + x = 5$

To use the elimination method, I have to line up the variables of the equations in standard form:

$y - 4x = 6$ Multiply the top equation by -3 → $-3y + 12x = -18$
$3y + x = 5$ $\underline{3y + \quad x = 5}$
$13x = -13 \rightarrow \quad \boldsymbol{x = -1}$

Now I can substitute the value of "*x*" into one of the equations to determine the value of "*y*"

$y - 4 * -1 = 6 \rightarrow \quad \boldsymbol{y = 2}$

I need to validate that I have the correct answers: $4 * -1 + 6 = 2$ √ and $3 * 2 - 1 = 5$ √

c)

$8 = 3r - s$
$r + 4s = 7$

To use the elimination method, I have to line up the variables of the equations in standard form:.

$3r - s = 8$ Multiply the top equation by 4 → $12r - 4s = 32$
$r + 4s = 7$ $\underline{\quad r + 4s = 7}$
$13r \quad = 39 \rightarrow \quad \boldsymbol{r = 3}$

Now I can substitute the value of "*r*" into one of the equations to determine the value of "*s*".

$3 + 4s = 7 \rightarrow \quad \boldsymbol{s = 1}$

I need to validate that I have the correct answers: $8 = 3 * 3 - 1$ √ and $3 + 4 * 1 = 7$ √

2) Using the elimination method, solve for the currents in our **initial** circuit.

$3I_1 + 2I_2 = 2$
$2I_1 + 5I_2 = 2$

These are already in standard form so I am going to multiply the top equation by -2.5.

$-7.5I_1 - 5I_2 = -5.$
$\underline{\quad 2I_1 + 5I_2 = 2}$
$-5.5I_1 = -3 \rightarrow \quad \boldsymbol{I_1 = 545\text{mA}}$

Now I will replace $\mathbf{I_1}$ with its value and solve for $\mathbf{I_2}$.

It does not matter which equation I use so I will use the first equation:

$3 * 545mA + 2I_2 = 2 \rightarrow \quad \boldsymbol{I_2 = 182\text{mA}}$

I need to validate that I have the correct answers:

$3 * 545mA + 2 * 182mA = 2$ √ and $2 * 545mA + 5 * 182mA = 2$ √

Problem Situation 5.6 – Determinants and Cramer's Rule

Yet another method for solving a system of equations is called **Cramer's Rule**. Cramer's rule is a *formula* for solving systems of equations in *matrix form* by using *determinants*.

A **Matrix** is a *rectangular* array of numbers arranged in *rows* and *columns*. Some examples are:

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \qquad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} \qquad \begin{vmatrix} a_1 \\ a_2 \\ a_3 \end{vmatrix} \qquad \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Size: 2 x 2 2 x 3 3 x 1 3 x 3

A **determinant** is a way to characterize a matrix by doing a mathematical operation, essentially cross-multiplying elements and adding or subtracting them. Determinants are very useful and but can only be defined with a *square* matrix, that is, an *equal* number of row and column elements. *Which of the matrixes above are square?*

To find the determinant, it is defined:

For a 2 x 2 matrix $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = (a_1 * b_2) - (a_2 * b_1)$

For a 3 x 3 matrix $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \begin{matrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{matrix} = (a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3) - (a_3b_2c_1 + b_3c_2a_1 + c_3a_2b_1)$

<u>EXAMPLE:</u>

$\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = (1 * 4) - (2 * 3) = 4 - 6 = -2$

1) Find the determinant for the following matrices:

a)
$\begin{vmatrix} -.24 & 1.54 \\ 2 & -6.2 \end{vmatrix} = (-0.24 * -6.2) - (2 * 1.54) = -1.952$

b)
$\begin{vmatrix} 1.4 & 2.6 \\ -8.9 & 5.4 \end{vmatrix} = (1.4 * 5.4) - (-8.9 * 2.6) = 30.7$

c)
$\begin{vmatrix} -6.1 & 3.1 \\ -3.4 & 16 \end{vmatrix} = (-6.1 * -16) - (-3.4 * 3.2) = -87.1$

d)
$\begin{vmatrix} 2 & 1 & -3 \\ 7 & 5 & 2 \\ -1 & 4 & 3 \end{vmatrix}$

$= [(2 * 5 * 3) + (1 * 2 * -1) + (-3 * 7 * 4)] - [(-1 * 5 * -3) + (4 * 2 * 2) + (3 * 7 * 1)]$
$= (30 - 3 - 84) - (15 + 16 + 21) = -108$

To use Cramer's Rule we first put the equations into *standard form*. We then 'group' the elements (coefficients) of the linear equations into different determinants, in a specific order. The order for a 2 x2 matrix is defined below. You find the determinants for each grouping and then do a simple division to solve for the two variables.

Cramer's Rule

$a_1x + b_1y = k_1$
$a_2x + b_2y = k_2$

$$x = \frac{\begin{vmatrix} k_1 & b_1 \\ k_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & k \\ a_2 & k_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

EXAMPLE:

$2x - y = -5$
$3x + 2y = 3$

$$x = \frac{\begin{vmatrix} -5 & -1 \\ 3 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}} = \frac{(-5 * 2) - (3 * -1)}{(2 * 2) - (3 * -1)} = \frac{-10 + 3}{4 + 3} = \frac{-7}{7} = -1$$

$$y = \frac{\begin{vmatrix} 2 & -5 \\ 3 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}} = \frac{(2 * 3) - (3 * -5)}{7} = \frac{6 + 15}{7} = \frac{21}{7} = 3$$

*Check by inserting into the original equations: 2 * -1 – 3 = -5 √ and 3*-1 + 2*3 = 3 √*

2) Using Cramer's Rule calculate the solution for the following systems of equations.

a)

$6 = 1.5x - y$
$4.5x + y = -6$

To use Cramer's Rule I need to put the equations in standard form.

$1.5x - y = 6$
$4.5x + y = -6$

$$x = \frac{\begin{vmatrix} 6 & -1 \\ -6 & 1 \end{vmatrix}}{\begin{vmatrix} 1.5 & -1 \\ 4.5 & 1 \end{vmatrix}} = \frac{(6 * 1) - (-6 * -1)}{(1.5 * 1) - (4.5 * -1)} = \frac{6 - 6}{1.5 + 4.5} = \frac{0}{6} = 0$$

$$y = \frac{\begin{vmatrix} 1.5 & 6 \\ 4.5 & -6 \end{vmatrix}}{\begin{vmatrix} 1.5 & -1 \\ 4.5 & 1 \end{vmatrix}} = \frac{(1.5 * -6) - (4.5 * 6)}{(1.5 * 1) - (4.5 * -1)} = \frac{-9 - 27}{1.5 + 4.5} = \frac{-36}{6} = -6$$

I need to validate that I have the correct answers. $6 = 1.5 * 0 + 6$ √ and $4.5 * 0 - 6 = -6$ √

b)

$2.3b = a - 2$
$a + b = 2$

To use Cramer's Rule I need to put the equations in standard form.

$a - 2.3b = 2$
$a + b = 2$

$$a = \frac{\begin{vmatrix} 2 & -2.3 \\ 2 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -2.3 \\ 1 & 1 \end{vmatrix}} = \frac{(2 * 1) - (2 * -2.3)}{(1 * 1) - (1 * -2.3)} = \frac{2 + 4.6}{1 + 2.3} = \frac{6.6}{3.3} = 2$$

$$b = \frac{\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & -2.3 \\ 1 & 1 \end{vmatrix}} = \frac{(1 * 2) - (1 * 2)}{(1 * 1) - (1 * -2.3)} = \frac{-2 - 2}{1 + 2.3} = \frac{0}{3.3} = 0$$

I need to validate that I have the correct answers: $2.3 * 0 = 2 - 2$ √ and $2 + 0 = 2$ √

c)

$3.4y = 3.8x - 4$
$6y + 4.9x = 6$

To use Cramer's Rule I need to put the equations in standard form.

$3.8x - 3.4y = 4$
$4.9x + 6y = 6$

$$x = \frac{\begin{vmatrix} 4 & -3.4 \\ 6 & 6 \end{vmatrix}}{\begin{vmatrix} 3.8 & -3.4 \\ 4.9 & 6 \end{vmatrix}} = \frac{(4 * 6) - (6 * -3.4)}{(3.8 * 6) - (4.9 * -3.4)} = \frac{24 + 4.6}{22.8 + 16.7} = \frac{28.6}{39.46} = 1.125$$

$$y = \frac{\begin{vmatrix} 3.8 & 4 \\ 4.9 & 6 \end{vmatrix}}{\begin{vmatrix} 3.8 & -3.4 \\ 4.9 & 6 \end{vmatrix}} = \frac{(3.8 * 6) - (4.9 * 4)}{(3.8 * 6) - (4.9 * -3.4)} = \frac{22.8 - 19.6}{22.8 + 16.7} = \frac{3.2}{39.46} = 0.0811$$

I need to validate that I have the correct answers.

$3.4 * 0.0811 = 3.8 * 1.125 - 4$ √ and $4.9 * 1.125 + 6 * 0.0811 = 6$ √

d)

$y = 4x - 6$
$6x - 2y = 7$

To use Cramer's Rule I need to put the equations in standard form.

$4x - y = 6$
$6x - 2y = 7$

$$x = \frac{\begin{vmatrix} 6 & -1 \\ 7 & -2 \end{vmatrix}}{\begin{vmatrix} 4 & -1 \\ 6 & -2 \end{vmatrix}} = \frac{(6 * -2) - (7 * -1)}{(4 * -2) - (6 * -1)} = \frac{-12 + 7}{-8 + 6} = \frac{-5}{-2} = 2.5$$

$$y = \frac{\begin{vmatrix} 4 & 6 \\ 6 & 7 \end{vmatrix}}{\begin{vmatrix} 4 & -1 \\ 6 & -2 \end{vmatrix}} = \frac{(4 * 7) - (6 * 6)}{(4 * -2) - (6 * -1)} = \frac{28 - 36}{-8 + 6} = \frac{-8}{-2} = 4$$

I need to validate that I have the correct answers: $4 = 4 * 2.5 - 6 \ \surd$ and $6 * 2.5 - 2 * 4 = 7 \ \surd$

3) Using Cramer's rule, solve for the currents in our **initial** circuit.

a)

$3I_1 + 2I_2 = 2$
$2I_1 + 5I_2 = 2$

This is already in standard form so I can directly use Cramer's Rule.

$$I_1 = \frac{\begin{vmatrix} 2 & 2 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix}} = \frac{(2 * 5) - (2 * 2)}{(3 * 5) - (2 * 2)} = \frac{10 - 4}{15 - 4} = \frac{6}{11} = 0.545$$

$$I_2 = \frac{\begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix}} = \frac{(3 * 2) - (2 * 2)}{(3 * 5) - (2 * 2)} = \frac{6 - 4}{15 - 4} = \frac{2}{11} = 0.182$$

I need to validate that I have the correct answers.

$$3 * 545mA + 2 * 182mA = 2 \ \surd \quad \text{and } 2 * 545mA + 5 * 182mA = 2 \ \surd$$

b) Use your calculator's equation solver to solve the same circuit. *Is it the same result? How much harder or easier was it?*

Every calculator will be a bit different. For the Casio 115: do this:

a) Press MODE 5 EQN to enter Equation Mode
b) Press 1
c) Use the Coefficient Editor to input the coefficients; make sure equations in standard form.
   i) Scroll using ^ and v)
   ii) After entering the number, press =
d) When you are finished entering coefficients, press =. This will display a solution. Every press of = will display another solution. You can scroll through the solutions.
e) Press AC to return to the Coefficient Editor.