Lesson	***Objectives***	***Material***
7.1	Introduction to Angles and Triangles	Angles, angles and more angles
7.2	Right Triangles – Pythagorean Theorem	Pythagoras legend
7.3	Pythagorean Theorem continued	Beach Walk
7.4	Introduction to the Trigonometric Functions	Soh Cah Toa
7.5	Ratios in Triangles	Similar Triangles
7.6	Trigonometry Functions	Pulling it all together

Prerequisite Assumptions

Before beginning the lesson, students should understand and be able to;

- ✓ Define polynomials
- ✓ Simplify polynomials
- ✓ Add and subtract polynomials
- ✓ Multiply polynomials
- ✓ Factor polynomials
- ✓ Determine solutions for second degree

Specific Objectives

By the end of this lesson, you should be able to;

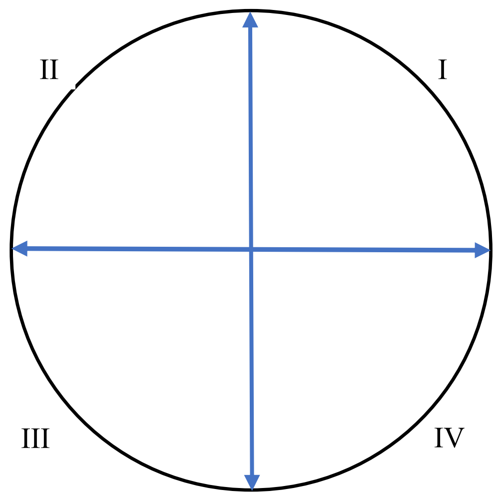
- ✓ Identify the hypotenuse, adjacent side, and opposite side of an acute angle in a right triangle.
- ✓ Determine the six trigonometric ratios for a given angle in a right triangle.
- ✓ Recognize the reciprocal relationship between sine/cosecant, cosine/secant, and tangent/cotangent.
- ✓ Use a calculator to find the value of the six trigonometric functions for any acute angle.
- ✓ Use a calculator to find the measure of an angle given the value of a trigonometric function.
- ✓ Use the Pythagorean Theorem to find the missing lengths of the sides of a right triangle.
- ✓ Find the missing lengths and angles of a right triangle.
- ✓ Solve applied problems using right triangle trigonometry.

Problem Situation 7.1 – Angles, Angles and more angles (Vocabulary)

Angle type .	**Angle size (degrees)**	**Angle size (radians)**
Acute	Between 0° and 90°	Between 0 rad and $\frac{\pi}{2}$ rad
Right	90°	$\frac{\pi}{2}$ rad
Straight	180∘	π rad
Obtuse	Between 180° and 360°	Between $\frac{\pi}{2}$ rad and 2π rad

A **phasor** is a line used to represent an electrical quantity as a *vector* having a *magnitude* and a *direction*. On a unit circle there are four (4) quadrants, starting from the positive x-axis and going counter-clockwise.

1) On the following diagram sketch a phasor with a 60° angle from the positive *x*-axis.
   a) Identify the type of angle this creates from the positive *x*-axis going in a counterclockwise direction.



   b) Identify the type of angle this creates from the positive *x*-axis going in a clockwise direction.

2) On the diagram sketch a phasor with a 180° angle.
   a) Identify the type of angle this creates from the positive *x*-axis going in a counterclockwise direction.
   b) Identify the type of angle this creates from the positive *x*-axis going in a clockwise direction.

3) On the diagram sketch a phasor with a -95° angle from the positive *x*-axis.
   a) Identify the type of angle this creates from the positive *x*-axis going in a counterclockwise direction.
   b) Identify the type of angle this creates from the positive *x*-axis going in a clockwise direction.

4) Identify the quadrant of each phasor with the angle;

   a) -34°
   b) 18°
   c) 97°
   d) 112°
   e) 286°
   f) -194°

5) Define a degree.

6) How many degrees in a complete circle?

7) Define a radius.

8) Sketch a circle with a radius of ~1.25 cm.

9) Determine the circumference of the circle.

Angles can be measured in *degrees* or in *radians*.

A **radian** (rad) is the *angle* made by taking the radius and wrapping it round the circumference of a circle. The radius of a circle can be laid out around the circle 2π times. Where $\pi \cong 3.1415$

$2\pi\, rads = 360°$ or $\pi\, rads = 180°$ so $1\, rads = \frac{360°}{2\pi} = 57.3°$

Conversion example: $142°$ is how many radians?

$142° * \frac{\pi\, rad}{180°} = 2.248\, rad$ or $142° * \frac{1\, rad}{57.3°} = 2.248\, rad$

10) Convert the following angles to radians

a) 90°

b) 45°

c) -60°

d) 67°

e) -34

f) 80°

11) Convert the following angles to degrees

a) $\frac{\pi}{6} rad$

b) $\pi\ rad$

c) $\frac{3\pi}{5} rad$

d) $\frac{\pi}{3} rad$

e) $1.2\pi\ rad$

f) $2.6\pi\ rad$

12) On the following diagram sketch a phasor with the following angles and indicate the quadrant in which the phasor is located.

a) $\frac{\pi}{6} rad$

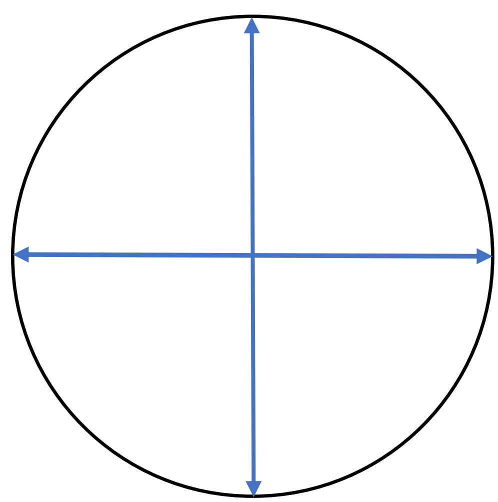
b) $\pi\ rad$

c) $\frac{3\pi}{5} rad$

d) $\frac{\pi}{3} rad$

e) $1.2\pi\ rad$

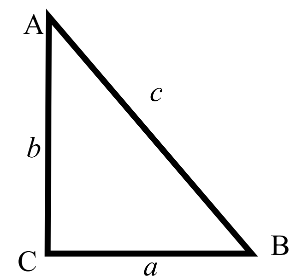
f) $2.6\pi\ rad$



Problem Situation 7.2 – Pythagoras legend

"The Pythagorean Theorem was one of the earliest theorems known to ancient civilizations. This famous theorem is named for the Greek mathematician and philosopher, Pythagoras. Pythagoras founded the Pythagorean School of Mathematics in Crotona, a Greek seaport in Southern Italy. He is credited with many contributions to mathematics although some of them may have actually been the work of his students."[1]

Right triangle – one angle is 90° (the right angle), designated $\measuredangle C$
Hypotenuse – the longest side of a right triangle and is opposite the right angle (90°, side c)
Pythagorean Theorem: $c^2 = a^2 + b^2$



The typical naming convention is to label the *sides*, a, b, c in lower case and the *angles* in upper case, $\measuredangle A, \measuredangle B, \measuredangle C$.
(Note: only the single letter for the angle name)

For all triangles, the angles sum to 180°. $\measuredangle A + \measuredangle B + \measuredangle C = 180°$

Determine the missing angle and sides.

1) When applying the Pythagorean theorem to a right triangle;
   a) How would you solve for side **c**, the hypotenuse?

   b) Determine the length of the hypotenuse, **c** when **a** = 12 cm, **b** = 19 cm.

2) When applying the Pythagorean theorem to a right triangle;
   a) How would you solve for side **b** given the hypotenuse, **c** and side **a**?

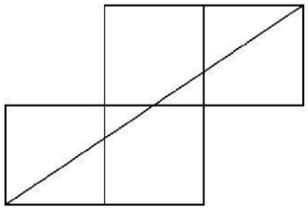
   b) Determine the length of side **b**, when the hypotenuse, **c** = 103 cm, **a** = 64 cm.

3) When applying the Pythagorean theorem to a right triangle;
   a) How would you solve for side **a** given the hypotenuse, **c** and side **b**?

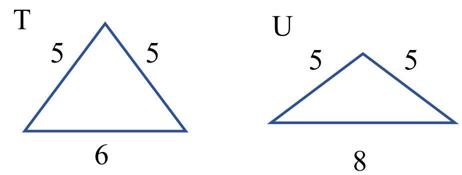
   b) Determine the length of side **a**, when the hypotenuse, **c** = 67 cm, **b** = 43 cm.

[1] http://jwilson.coe.uga.edu/emt669/student.folders/morris.stephanie/emt.669/essay.1/pythagorean.html

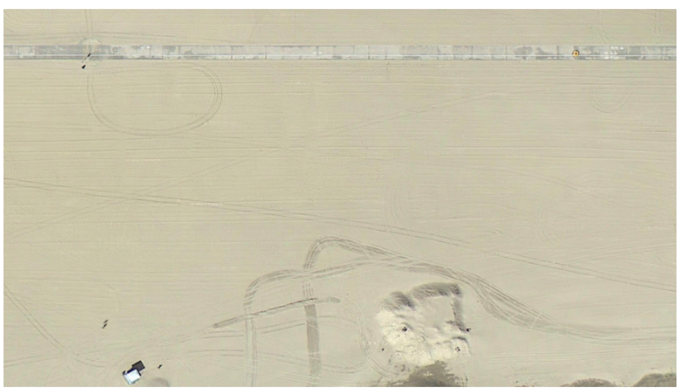
4) Four - 6 cm squares are placed edge to edge as shown below.
   What is the length of the diagonal line as drawn?



5) The triangle T has sides of length 6”, 5”, 5”. The triangle U has sides of length 8”, 5”, and 5”.
   What is the ratio of the area of T to the area of U (**area T : area U**)



<u>Problem Situation 7.3 – Beach Walk</u>



Dan Meyer

1) Who gets to the taco cart first? Take a guess.

2) What information do you need?

3) Who gets to the cart first?

Problem Situation 7.4 – Soh Cah Toa

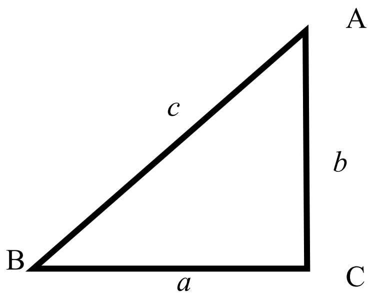
Trigonometry is simply the *art of measuring* of a triangle. For this lesson we are only talking about a right triangle.

For the right triangle as shown

$side\ a\ is\ opposite\ \measuredangle A$

$side\ b\ is\ opposite\ \measuredangle B$

$side\ c, the\ hypotenuse\ is\ opposite\ \measuredangle C$



$$Sin(\theta) = \frac{opposite}{hypotenuse} \rightarrow Sin(\measuredangle A) = \frac{a}{c} \rightarrow Sin(\measuredangle B) = \frac{b}{c}$$

Sin = **O**pposite / **H**ypotenuse (Soh)

Example:

Side a = 34.2 m and $\measuredangle A$ = 36°, determine the hypotenuse.

$$Sin(\theta) = \frac{opposite}{hypotenuse} \rightarrow Sin(\measuredangle A) = \frac{a}{c} \rightarrow Sin(\measuredangle B) = \frac{b}{c}$$

$$Sin(36°) = \frac{34.2}{c} \rightarrow c * Sin(36°) = 34.2 \rightarrow c = \frac{34.2}{Sin(36°)} \rightarrow c = 58.2\ m$$

$$Cos(\theta) = \frac{adjacent}{hypotenuse} \rightarrow Cos(\measuredangle A) = \frac{b}{c} \rightarrow Cos(\measuredangle B) = \frac{a}{c}$$

Cos = **A**djacent / **H**ypotenuse (Cah)

Example:

Side c = 58.2 m and $\measuredangle B$ = 54°, determine side a

$$Cos(\theta) = \frac{adjacent}{hypotenuse} \rightarrow Cos(\measuredangle B) = \frac{a}{c}$$

$$Cos(54°) = \frac{a}{58.2} \rightarrow 58.2 * Cos(54°) = a \rightarrow a = 34.2\ m$$

$$Tan(\theta) = \frac{opposite}{adjacent} \rightarrow Tan(\measuredangle A) = \frac{a}{b} \rightarrow Tan(\measuredangle B) = \frac{b}{a}$$

Tan = **O**pposite / **A**djacent (Toa)

Example:

Side a = 34.2 m and $\measuredangle B$ = 54°, determine side b

$$Tan(\theta) = \frac{opposite}{adjacent} \rightarrow Tan(\measuredangle B) = \frac{b}{a}$$

$$Tan(54°) = \frac{b}{34.2} \rightarrow 34.2 * Tan(54°) = b \rightarrow b = 47.1\ m$$

1) Determine the requested piece of data for each right triangle.

   a) Side b = 293 mm and $\measuredangle B$ = 21°, determine the hypotenuse.

   b) Side a = 310 m and $\measuredangle A$ = 66°, determine the hypotenuse.

   c) Side c = 21 m and $\measuredangle A$ = 70°, determine side b.

   d) Side a = 185 ft and $\measuredangle B$ = 43°, determine side c.

   e) Side a = 88 cm and $\measuredangle A$ = 14°, determine side b.

   f) Side b = 109 in and $\measuredangle B$ = 25°, determine side a.

Problem Situation 7.4 - The inverse trigonometric functions

We have used **inverse** operations several times this semester. For example, addition and subtraction are *inverse* operations; and multiplication and division are *inverse* operations. Each operation does the *opposite* of its inverse. We use the same idea in trigonometry.

Inverse trig functions do the opposite of the "regular" trig functions.

$$Sin(\theta) = \frac{opposite}{hypotenuse} \rightarrow Sin^{-1}\left(\frac{opposite}{hypotenuse}\right) = \theta \; (often\ called\ arcsin)$$

$$Cos(\theta) = \frac{adjacent}{hypotenuse} \rightarrow Cos^{-1}\left(\frac{adjacent}{hypotenuse}\right) = \theta\ \theta \; (often\ called\ arccos)$$

$$Tan(\theta) = \frac{opposite}{adjacent} \rightarrow Tan^{-1}\left(\frac{opposite}{adjacent}\right) = \theta\ \theta \; (often\ called\ arctan)$$

Example:

Side **a** = 34.2 m and **c** = 58.2 m

$$Sin(\measuredangle A) = \frac{opposite}{hypotenuse} \rightarrow Sin(\measuredangle A) = \frac{a}{c} \rightarrow Sin^{-1}\left(\frac{a}{c}\right) = \measuredangle A$$

$$\measuredangle A = Sin^{-1}\left(\frac{34.2}{58.2}\right) \rightarrow \measuredangle A = 36°$$

1) Determine the requested piece of data for each right triangle.

   a) Side **c** = 254 m and side **b** = 133m, determine $\measuredangle \mathbf{B}$.

   b) Side **b** = 227.2 cm and side **a** = 125.4 cm, determine $\measuredangle \mathbf{B}$.

   c) Side **c** = 138 in and side **b** = 55.8 in, determine $\measuredangle \mathbf{A}$.

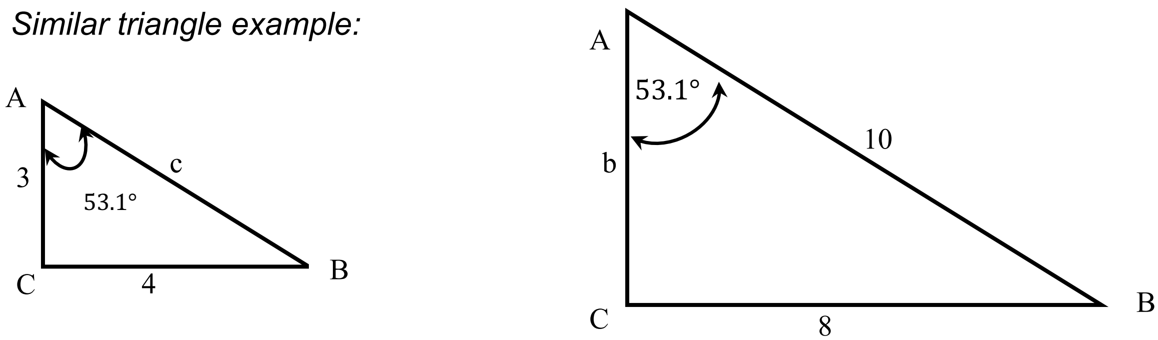
   d) Side **c** = 110 ft and side **b** = 33.9 ft, determine $\measuredangle \mathbf{B}$.

   e) Side **a** = 229 m and side **b** = 98.9 m, determine $\measuredangle \mathbf{A}$**.**

Problem Situation 7.5 – Similar Triangles

Similar Triangles are two triangles that have *equal corresponding angles* with *corresponding sides in the same proportion*.

Similar triangle example:



Example:

Determine $\measuredangle B$ using Similar Triangles:

$\measuredangle B = 180° - \measuredangle C - \measuredangle A = 180° - 90° - 53.1° \rightarrow \measuredangle B = 36.9°$

Determine side c of the small triangle using Similar Triangles:

$$\frac{small\ a}{large\ a} = \frac{small\ b}{large\ b} = \frac{small\ c}{large\ c}$$

$$\frac{4}{8} = \frac{3}{large\ b} \rightarrow large\ b = 6 \qquad \frac{4}{8} = \frac{small\ c}{10} \rightarrow small\ c = 5$$

1) A LED screen that you are programming is 3 cm by 5 cm. You must upscale to a much larger screen size with a diagonal measurement of 12 cm. Determine the ratio large screen side : small screen side. Also determine the width and length of the larger screen.

2) Determine the missing angle and sides for the pair of similar triangles.
$\measuredangle C = 94°\ and\ \measuredangle D = 68°$

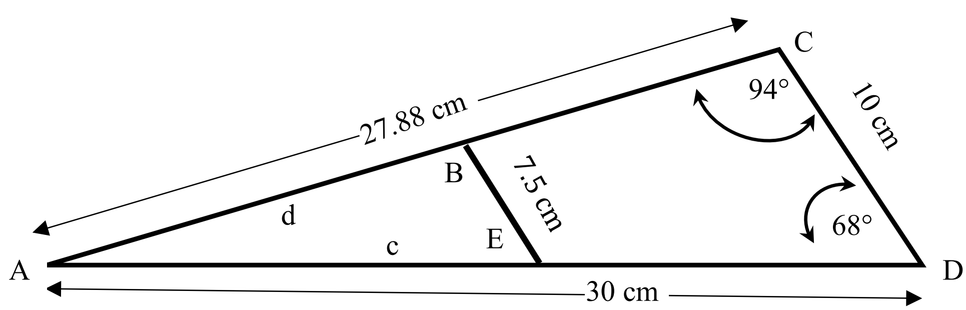
$\measuredangle A =$

$\measuredangle B =$

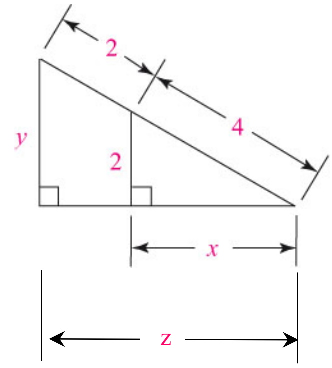
$\measuredangle E =$

$c =$

$d =$



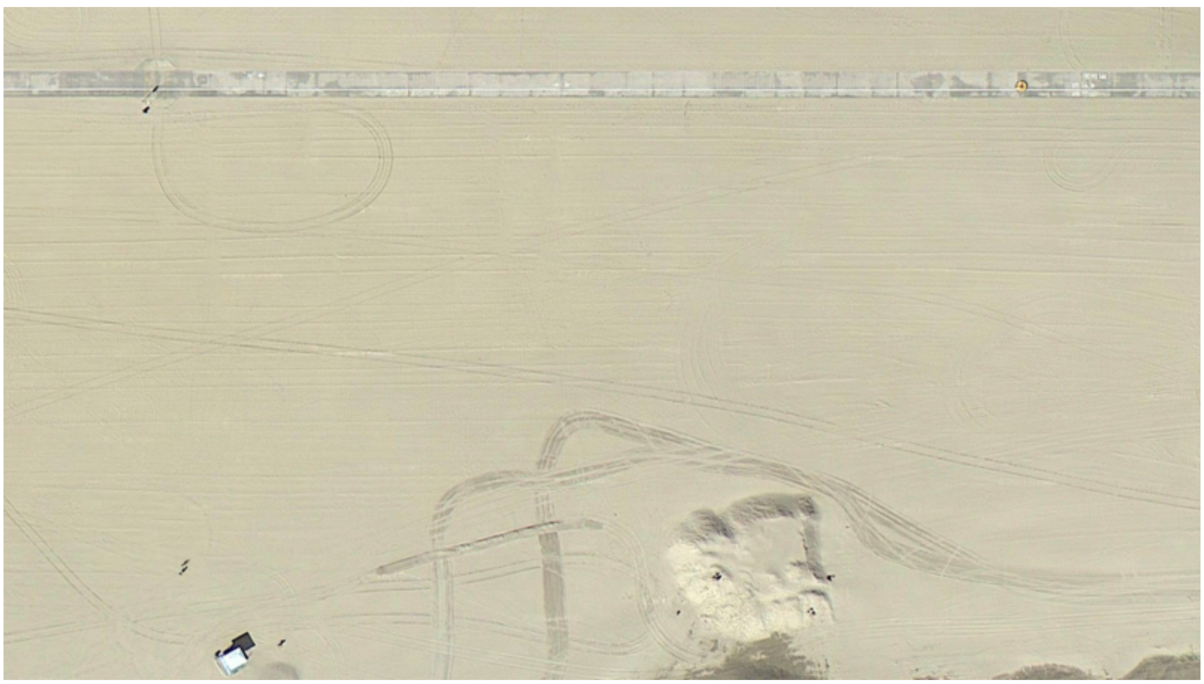
3) Determine the missing sides of the similar triangles.



x = _______ *y* = __________ z = __________

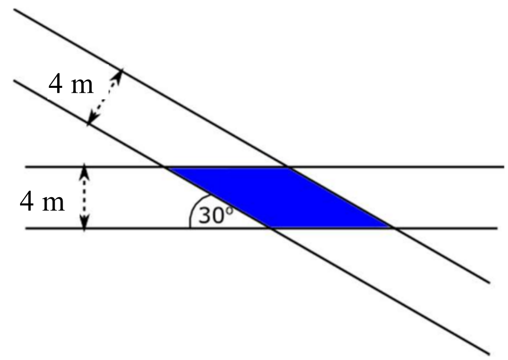
Problem Situation 7.6 – Pulling it all together

4) Where would the taco cart have to be so that both people would reach it at the same time? Draw the point where you think the taco cart should be.

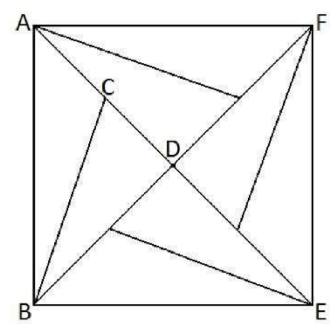


5) Determine the optimum placement of the taco cart and the time it would take Dan and Ben to walk to it.

6) Overlapping roads, each of width 4 meters, are laid across each other at an angle of 30°, as shown in the diagram. Determine the area of the overlap.



7) The diagram has a rotational symmetry of the order of 4 about D. If $\measuredangle A$BC is 15° and the area of ABEF is 24 cm2, what is the length of CD?



8) The diagram depicts a diamond ring with the diagonals measuring 6 mm x 8 mm. Determine the radius of the diamond.