COURSE COMPETENCIES

1. **Solve equations using right angle trigonometry**
   - You use sine, cosine, and tangent ratios to compute sides and/ or angles of right triangles
2. **Solve right triangles**
   - You solve for all angles in a right triangle.
   - You use the Pythagorean theorem to compute a side of a right triangle
   - You use sine, cosine, and tangent ratios to compute sides and/ or angles of right triangles

BACKGROUND

This module introduces right triangle trigonometry. The students need to start developing a routine expertise in applying these functions.

EXPLICIT CONNECTIONS

It is important that each person understands the link between these trigonometric functions and the electronic applications.

NOTES TO SELF

- Encourage each student to check his or her answers. They just do not want to take the time to check their answers.

Duration Minutes	Lesson	Suggested Structure
15	Lecture - Introduction to Angles and Triangles	Cohort
15	Problem Situation 7.1 – Angles, Angles and more angles	Group
10	Blackboard: Practice Set 1 - Angles and Units	Individual
15	Lecture - Pythagorean's Theorem	Cohort
15	Problem Situation 7.2 – Pythagoras legend	Group
15	Problem Situation 7.3 – Beach Walk	Group
10	Blackboard: Practice Set 2 - Pythagorean	Individual
15	Lecture - Introduction to Trigonometric Functions	Cohort
15	Problem Situation 7.4 – Soh Cah Toa	Group
15	Lecture - Inverse Trigonometric Functions	Cohort
15	Problem Situation 7.4 - The inverse trigonometric functions	Group
15	Blackboard: Practice Set 3 - Trig Functions	Individual
15	Blackboard: Practice Set 4 - Inverse Trig Functions	Individual
15	Problem Situation 7.5 – Similar Triangles	Group
15	Blackboard: Practice Set 3 - Trig Functions	Individual
15	Blackboard: Practice Set 4 - Inverse Trig Functions	Individual
15	Problem Situation 7.5 – Similar Triangles	Group
20	Problem Situation 7.6 – Pulling it all together	Group
20	Quiz	Cohort
15	Excel	Cohort

Lesson	***Objectives***	***Material***
7.1	Introduction to Angles and Triangles	Angles, angles and more angles
7.2	Right Triangles – Pythagorean Theorem	Pythagoras legend
7.3	Pythagorean Theorem continued	Beach Walk
7.4	Introduction to the Trigonometric Functions	Soh Cah Toa
7.5	Ratios in Triangles	Similar Triangles
7.6	Trigonometry Functions	Pulling it all together

Prerequisite Assumptions

Before beginning the lesson, students should understand and be able to;

- ✓ Define polynomials
- ✓ Simplify polynomials
- ✓ Add and subtract polynomials
- ✓ Multiply polynomials
- ✓ Factor polynomials
- ✓ Determine solutions for second degree

Specific Objectives

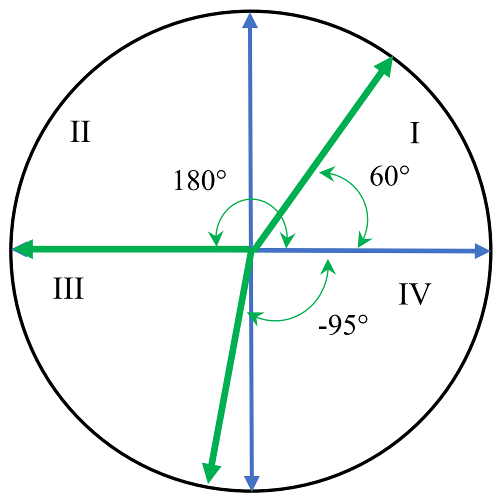
By the end of this lesson, you should be able to;

- ✓ Identify the hypotenuse, adjacent side, and opposite side of an acute angle in a right triangle.
- ✓ Determine the six trigonometric ratios for a given angle in a right triangle.
- ✓ Recognize the reciprocal relationship between sine/cosecant, cosine/secant, and tangent/cotangent.
- ✓ Use a calculator to find the value of the six trigonometric functions for any acute angle.
- ✓ Use a calculator to find the measure of an angle given the value of a trigonometric function.
- ✓ Use the Pythagorean Theorem to find the missing lengths of the sides of a right triangle.
- ✓ Find the missing lengths and angles of a right triangle.
- ✓ Solve applied problems using right triangle trigonometry.

Problem Situation 7.1 – Angles, Angles and more angles (Vocabulary)

Angle type	Angle size (degrees)	Angle size (radians)
Acute	Between 0° and 90°	Between 0 rad and $\frac{\pi}{2}$ rad
Right	90°	$\frac{\pi}{2}$ rad
Straight	180∘	π rad
Obtuse	Between 90° and 180°	Between $\frac{\pi}{2}$ rad and π rad

A **phasor** is a line used to represent an electrical quantity as a *vector* having a *magnitude* and a *direction*. On a unit circle there are four (4) quadrants, starting from the positive x-axis and going counter-clockwise.



1) On the diagram sketch a phasor with a 60° angle from the positive *x*-axis.
   a) Identify the type of angle this creates from the positive *x*-axis going in a counterclockwise direction.

This creates an acute angle from the positive x-axis going in a counterclockwise direction.

   b) Identify the type of angle this creates from the positive *x*-axis going in a clockwise direction.

This creates an obtuse angle from the positive x-axis going in a clockwise direction.

2) On the diagram sketch a phasor with a 180° angle.
   a) Identify the type of angle this creates from the positive *x*-axis going in a counterclockwise direction.

This creates a straight angle from the positive x-axis going in a counterclockwise direction.

   b) Identify the type of angle this creates from the positive *x*-axis going in a clockwise direction.

This creates a straight angle from the positive x-axis going in a clockwise direction.

3) On the diagram sketch a phasor with a -95° angle from the positive *x*-axis.
   a) Identify the type of angle this creates from the positive *x*-axis going in a counterclockwise direction.

This creates an obtuse angle from the positive x-axis going in a counterclockwise direction.

   b) Identify the type of angle this creates from the positive *x*-axis going in a clockwise direction.

This creates an obtuse angle from the positive x-axis going in a counterclockwise direction.

4) Identify the quadrant of each phasor with the angle;

   a) -34° is in Q4

   b) 18° is in Q1

   c) 97° is in Q2

   d) 112° is in Q2

   e) 286° is in Q4

   f) -194° is in Q2

5) Define a degree.

$A\ degree\ is\ \frac{1}{360}\ of\ a\ complete\ rotation\ of\ a\ circle.$

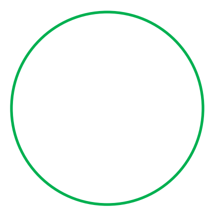
6) How many degrees in a complete circle?

There are 360° in a complete circle.

7) Define a radius.

A radius is a straight line from the center of a circle to its radius.

8) Sketch a circle with a radius of ~1.25 cm.



9) Determine the circumference of the circle.

To determine the circumference, I would use the geometric formula,

$$C = 2\pi \mathrm{r} = 2\pi * 1.25cm = 7.854\ cm$$

Angles can be measured in *degrees* or in *radians*.

A **radian** (rad) is the *angle* made by taking the radius and wrapping it round the circumference of a circle. The radius of a circle can be laid out around the circle 2π times. Where $\pi \cong 3.1415$

$2\pi\ rads = 360°$ or $\pi\ rads = 180°$ so $1\ rads = \frac{360°}{2\pi} = 57.3°$

Conversion example: $142°$ is how many radians?

$142° * \frac{\pi\ rad}{180°} = 2.248\ rad$ or $142° * \frac{1\ rad}{57.3°} = 2.248\ rad$

10) Convert the following angles to radians

a) 90° $= \frac{\pi}{2} rad = 1.57 rad$

b) 45° $= \frac{\pi}{4} rad = 0.785 rad$

c) -60° $= -\frac{\pi}{3} rad = -1.047 rad$

d) 67° $= 1.17 rad$

e) -34 $= -0.5934 rad$ °

f) 80° $= 1.396 rad$

11) Convert the following angles to degrees

a) $\frac{\pi}{6} rad = 30°$

b) $\pi\ rad = 180°$

c) $\frac{3\pi}{5} rad = 108°$

d) $\frac{\pi}{3} rad = 60°$

e) $1.2\pi\ rad = 216°$

f) $2.6\pi\ rad = 468°$

12) On the following diagram sketch a phasor with the following angles and indicate the quadrant in which the phasor is located.

a) $\frac{\pi}{6} rad$ in Q1

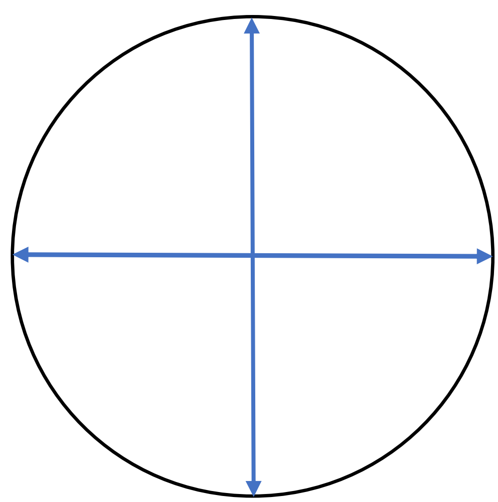
b) $\pi\ rad$ is between Q2 and Q3

c) $\frac{3\pi}{5} rad$ in Q2

d) $\frac{\pi}{3} rad$ in Q1

e) $1.2\pi\ rad$ in Q3

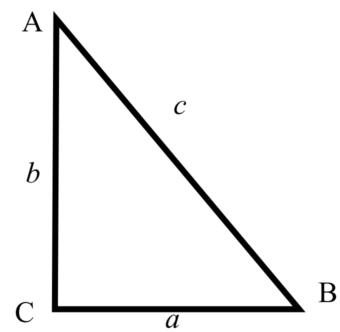
f) $2.6\pi\ rad$ in Q2



Problem Situation 7.2 – Pythagoras legend

> "The Pythagorean Theorem was one of the earliest theorems known to ancient civilizations. This famous theorem is named for the Greek mathematician and philosopher, Pythagoras. Pythagoras founded the Pythagorean School of Mathematics in Crotona, a Greek seaport in Southern Italy. He is credited with many contributions to mathematics although some of them may have actually been the work of his students."[1]

Right triangle – one angle is 90° (the right angle), designated $\measuredangle C$
Hypotenuse – the longest side of a right triangle and is opposite the right angle (90°, side c)
Pythagorean Theorem: $c^2 = a^2 + b^2$



The typical naming convention is to label the *sides*, a, b, c in lower case and the *angles* in upper case, $\measuredangle A, \measuredangle B, \measuredangle C$.
(Note: only the single letter for the angle name)

For all triangles, the angles sum to 180°. $\measuredangle A + \measuredangle B + \measuredangle C = 180°$

Determine the missing angle and sides.

1) When applying the Pythagorean theorem to a right triangle;
   a) How would you solve for side **c**, the hypotenuse?

$c^2 = a^2 + b^2 \rightarrow c = \sqrt{a^2 + b^2}$

   b) Determine the length of the hypotenuse, **c** when **a** = 12 cm, **b** = 19 cm.

$c = \sqrt{12cm^2 + 19cm^2} = 22.5\ cm$

2) When applying the Pythagorean theorem to a right triangle;
   a) How would you solve for side **b** given the hypotenuse, **c** and side **a**?

$c^2 = a^2 + b^2 \rightarrow b = \sqrt{c^2 - a^2}$

   b) Determine the length of side **b**, when the hypotenuse, **c** = 103 cm, **a** = 64 cm.

$b = \sqrt{103cm^2 - 64cm^2} = 80.7cm$

3) When applying the Pythagorean theorem to a right triangle;
   a) How would you solve for side **a** given the hypotenuse, **c** and side **b**?

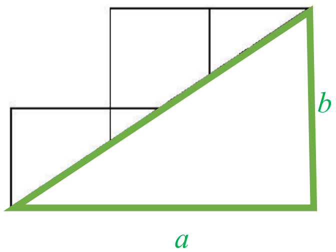
$c^2 = a^2 + b^2 \rightarrow a = \sqrt{c^2 - b^2}$

   b) Determine the length of side **a**, when the hypotenuse, **c** = 67 cm, **b** = 43 cm.

$a = \sqrt{67cm^2 - 53cm^2} = 51.4cm$

[1] http://jwilson.coe.uga.edu/emt669/student.folders/morris.stephanie/emt.669/essay.1/pythagorean.html

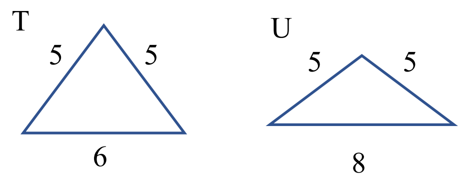
4) Four - 6 cm squares are placed edge to edge as shown below
What is the length of the diagonal line as drawn?



This forms a right triangle with $a = 3 * 6cm = 18cm \ and \ b = 2 * 6cm = 12cm.$

$I \ know \ that \ c = \sqrt{a^2 + b^2} = \sqrt{18^2 + 12^2} = 21.6cm.$

5) The triangle T has sides of length 6", 5", 5". The triangle U has sides of length 8", 5", and 5".
What is the ratio of the area of T to the area of U (**area T : area U**)

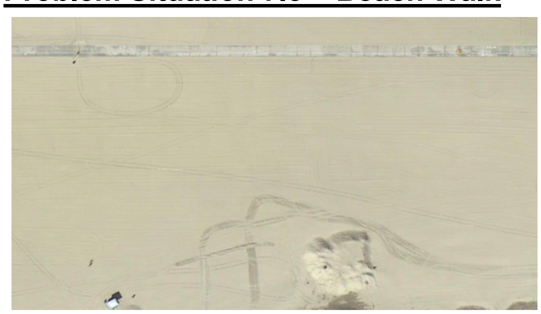


Area of a right triangle is ½ height*base. A=½hb
Triangle T: $h_T = \sqrt{5^2 - 3^2} = 4"$
$A_T = 2 * \frac{1}{2} * 4*3 = 12 \ sq \ in$

Triangle U: $h_T = \sqrt{5^2 - 4^2} = 3"$
$A_U = 2 * \frac{1}{2} * 3*4 = 12 \ sq \ in$

The ratio T:U is 12:12 or 1:1

Problem Situation 7.3 – Beach Walk



Dan Meyer
1) Who gets to the taco cart first? Take a guess.
Show Act 1

2) What information do you need?

Show Act 2 Dimensions and Act 2 Speed

3) Who gets to the cart first?
Looking at the right triangle, Dan travels side a and b.

$Dan \ walked \ {}^{2ft}/_{sec} \ for \ 325.6 \ ft \ and \ 5 \ {}^{ft}/_{sec} \ for \ 562.6 \ ft$

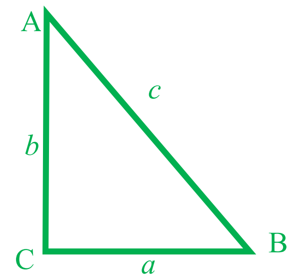
$$\frac{325.6\,\cancel{ft}}{1} * \frac{sec}{2\cancel{ft}} + \frac{562.6\,\cancel{ft}}{1} * \frac{sec}{5\cancel{ft}} = 275.3sec$$

$$= 4\,min\ and\ 35sec$$

$$Ben\ walked\ {}^{2ft}/_{sec}\ for\ \sqrt{325.6\,ft^2 + 562.6\,ft^2}$$

$$\frac{650\,\cancel{ft}}{1} * \frac{sec}{2\cancel{ft}} = 325sec = 5\,min\ and\ 25sec$$

Show Act 3.



Problem Situation 7.4 – Soh Cah Toa

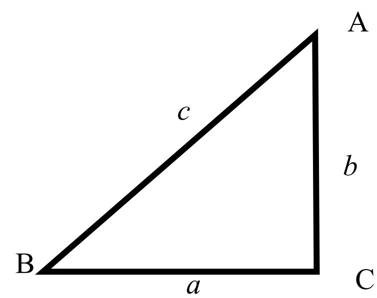
Trigonometry is simply the *art of measuring* of a triangle. For this lesson we are only talking about a right triangle.

For the right triangle as shown

$side\ a\ is\ opposite\ \measuredangle A$

$side\ b\ is\ opposite\ \measuredangle B$

$side\ c, the\ hypotenuse\ is\ opposite\ \measuredangle C$



$$Sin(\theta) = \frac{opposite}{hypotenuse} \rightarrow Sin(\measuredangle A) = \frac{a}{c} \rightarrow Sin(\measuredangle B) = \frac{b}{c}$$

Sin = **O**pposite / **H**ypotenuse (Soh)

Example:

Side a = 34.2 m and $\measuredangle A$ = 36°, determine the hypotenuse.

$$Sin(\theta) = \frac{opposite}{hypotenuse} \rightarrow Sin(\measuredangle A) = \frac{a}{c} \rightarrow Sin(\measuredangle B) = \frac{b}{c}$$

$$Sin(36°) = \frac{34.2}{c} \rightarrow c * Sin(36°) = 34.2 \rightarrow c = \frac{34.2}{Sin(36°)} \rightarrow c = 58.2\,m$$

$$Cos(\theta) = \frac{adjacent}{hypotenuse} \rightarrow Cos(\measuredangle A) = \frac{b}{c} \rightarrow Cos(\measuredangle B) = \frac{a}{c}$$

Cos = **A**djacent / **H**ypotenuse (Cah)

Example:

Side c = 58.2 m and $\measuredangle B$ = 54°, determine side a

$$Cos(\theta) = \frac{adjacent}{hypotenuse} \rightarrow Cos(\measuredangle B) = \frac{a}{c}$$

$$Cos(54°) = \frac{a}{58.2} \rightarrow 58.2 * Cos(54°) = a \rightarrow a = 34.2\,m$$

$$Tan(\theta) = \frac{opposite}{adjacent} \rightarrow Tan(\measuredangle A) = \frac{a}{b} \rightarrow Tan(\measuredangle B) = \frac{b}{a}$$

Tan = **O**pposite / **A**djacent (Toa)

<u>*Example:*</u>

Side a = 34.2 m and ∡B = 54°, determine side b

$$Tan(\theta) = \frac{opposite}{adjacent} \rightarrow Tan(\measuredangle B) = \frac{b}{a}$$

$$Tan(54°) = \frac{b}{34.2} \rightarrow 34.2 * Tan(54°) = b \rightarrow b = 47.1\,m$$

1) Determine the requested piece of data for each right triangle.

a) Side b = 293 mm and ∡B = 21°, determine the hypotenuse.

I know ∡B and the opposite side so I will use the Sine function to determine the hypotenuse.

$$Sin(\measuredangle B) = \frac{Opposite}{Hypotenuse} = \frac{b}{c} \rightarrow Sin(\measuredangle 21°) = \frac{293mm}{c} \rightarrow c = \frac{293mm}{Sin(\measuredangle 21°)} = 818mm$$

b) Side a = 310 m and ∡A = 66°, determine the hypotenuse.

I know ∡A and the opposite side so I will use the Sine function to determine the hypotenuse.

$$Sin(\measuredangle A) = \frac{Opposite}{Hypotenuse} = \frac{a}{c} \rightarrow Sin(\measuredangle 66°) = \frac{310m}{c} \rightarrow c = \frac{310m}{Sin(\measuredangle 66°)} = 340m$$

c) Side c = 21 m and ∡A = 70°, determine side b.

I know ∡A and the hypotenuse so I will use the Cos function to determine side b.

$$Cos(\measuredangle A) = \frac{Adjacent}{Hypotenuse} = \frac{b}{c} \rightarrow Cos(\measuredangle 70°) = \frac{b}{21m} \rightarrow b = Cos(\measuredangle 70°) * 21m = 7.18m$$

d) Side a = 185 ft and ∡B = 43°, determine side c.

I know ∡B and the adjacent side so I will use the Cos function to determine side c.

$$Cos(\measuredangle B) = \frac{Adjacent}{Hypotenuse} = \frac{a}{c} \rightarrow Cos(\measuredangle 43°) = \frac{185ft}{c} \rightarrow c = \frac{185ft}{Cos(\measuredangle 43°)} = 253ft$$

e) Side a = 88 cm and ∡A = 14°, determine side b.

I know ∡A and the opposite side so I will use the Tan function to determine the adjacent side.

$$Tan(\measuredangle A) = \frac{Opposite}{Adjacent} = \frac{a}{b} \rightarrow Tan(\measuredangle 14°) = \frac{88cm}{b} \rightarrow b = \frac{88cm}{Tan(\measuredangle 14°)} = 353cm$$

f) Side b = 109 in and ∡B = 25°, determine side a.

I know ∡B and the opposite side so I will use the Tan function to determine the adjacent side.

$$Tan(\measuredangle B) = \frac{Opposite}{Adjacent} = \frac{b}{a} \rightarrow Tan(\measuredangle 25°) = \frac{109in}{a} \rightarrow b = \frac{109in}{Tan(\measuredangle 25°)} = 234cm$$

Problem Situation 7.4 - The inverse trigonometric functions

We have used **inverse** operations several times this semester. For example, addition and subtraction are *inverse* operations; and multiplication and division are *inverse* operations. Each operation does the *opposite* of its inverse. We use the same idea in trigonometry.

Inverse trig functions do the opposite of the “regular” trig functions.

$$Sin(\theta) = \frac{opposite}{hypotenuse} \rightarrow Sin^{-1}\left(\frac{opposite}{hypotenuse}\right) = \theta\ (often\ called\ arcsin)$$

$$Cos(\theta) = \frac{adjacent}{hypotenuse} \rightarrow Cos^{-1}\left(\frac{adjacent}{hypotenuse}\right) = \theta\ \theta\ (often\ called\ arccos)$$

$$Tan(\theta) = \frac{opposite}{adjacent} \rightarrow Tan^{-1}\left(\frac{opposite}{adjacent}\right) = \theta\ \theta\ (often\ called\ arctan)$$

Example:
Side **a** = 34.2 m and **c** = 58.2 m

$$Sin(\measuredangle A) = \frac{opposite}{hypotenuse} \rightarrow Sin(\measuredangle A) = \frac{a}{c} \rightarrow Sin^{-1}\left(\frac{a}{c}\right) = \measuredangle A$$

$$\measuredangle A = Sin^{-1}\left(\frac{34.2}{58.2}\right) \rightarrow \measuredangle A = 36°$$

1) Determine the requested piece of data for each right triangle.

a) Side **c** = 254 m and side **b** = 133m, determine $\measuredangle \mathbf{B}$.

I know side b, and the hypotenuse so determine∡B I will use the Sine function.

$$Sin(\measuredangle B) = \frac{Opposite}{Hypotenuse} = \frac{b}{c} \rightarrow Sin(\measuredangle B) = \frac{133m}{254m} \rightarrow \measuredangle B = Sin^{-1}\left(\frac{133m}{254m}\right) = 31.7°$$

b) Side **b** = 227.2 cm and side **a** = 125.4 cm, determine $\measuredangle \mathbf{B}$.

I know side b and side a so to determine ∡B I will use the Tan function.

$$Tan(\measuredangle B) = \frac{Opposite}{Adjacent} = \frac{b}{a} \rightarrow Tan(\measuredangle B) = \frac{227.2cm}{125.4cm} \rightarrow \measuredangle B = Tan^{-1}\left(\frac{227.2cm}{125.4cm}\right) = 61.1°$$

c) Side **c** = 138 in and side **b** = 55.8 in, determine $\measuredangle \mathbf{A}$.

I know side b and the hypotenuse so to determine ∡A I will use the Cos function.

$$Cos(\measuredangle A) = \frac{Adjacent}{Hypotenuse} = \frac{b}{c} \rightarrow Tan(\measuredangle B) = \frac{55.8in}{138in} \rightarrow \measuredangle B = Tan^{-1}\left(\frac{55.8in}{138in}\right) = 66.2°$$

d) Side **c** = 254 m and side **b** = 133 m, determine $\measuredangle$**A.**

$I\ know\ side\ b, and\ the\ hypotenuse\ so\ determine \measuredangle B\ \ I\ will\ use\ the\ Cos\ function.$

$$Cos(\measuredangle A) = \frac{Adjacent}{Hypotenuse} = \frac{b}{c} \rightarrow Cos(\measuredangle A) = \frac{133m}{254m} \rightarrow \measuredangle A = Sin^{-1}\left(\frac{133.4m}{253.9m}\right) = 68.3°$$

e) Side **c** = 110 ft and side **b** = 33.9 ft, determine $\measuredangle$**B**.

$I\ know\ side\ b, and\ the\ hypotenuse\ so\ determine \measuredangle B\ \ I\ will\ use\ the\ Sine\ function.$

$$Sin(\measuredangle B) = \frac{Opposite}{Hypotenuse} = \frac{b}{c} \rightarrow Sin(\measuredangle B) = \frac{110ft}{33.9ft} \rightarrow \measuredangle B = Sin^{-1}\left(\frac{110ft}{33.9ft}\right) = 18°$$

f) Side **a** = 229 m and side **b** = 98.9 m, determine $\measuredangle$**A**.

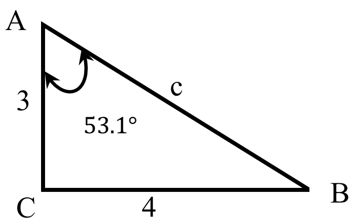
$I\ know\ side\ b\ and\ side\ a\ so\ to\ determine\ \measuredangle A\ \ I\ will\ use\ the\ Tan\ function.$

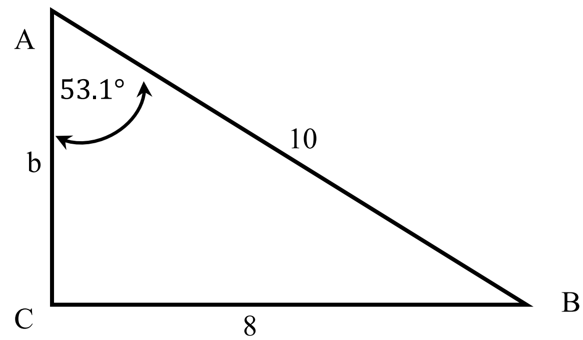
$$Tan(\measuredangle A) = \frac{Opposite}{Adjacent} = \frac{a}{b} \rightarrow Tan(\measuredangle A) = \frac{229m}{98.9m} \rightarrow \measuredangle A = Tan^{-1}\left(\frac{229m}{98.9m}\right) = 66.6°$$

<u>Problem Situation 7.5 – Similar Triangles</u>

Similar Triangles are two triangles that have *<u>equal</u> corresponding angles* with *corresponding sides in the same <u>proportion</u>*.

Similar triangle example:





Example:

Determine $\measuredangle B$ using Similar Triangles:

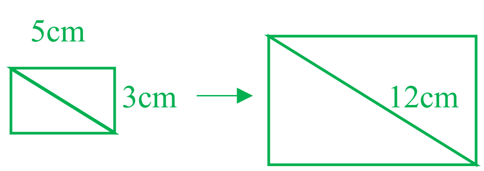
$\measuredangle B = 180° - \measuredangle C - \measuredangle A = 180° - 90° - 53.1° \rightarrow \measuredangle B = 36.9°$

Determine side c of the small triangle using Similar Triangles:

$$\frac{small\ a}{large\ a} = \frac{small\ b}{large\ b} = \frac{small\ c}{large\ c}$$

$$\frac{4}{8} = \frac{3}{large\ b} \rightarrow large\ b = 6 \qquad \frac{4}{8} = \frac{small\ c}{10} \rightarrow small\ c = 5$$

1) A LED screen that you are programming is 3 cm by 5 cm. You must upscale to a much larger screen size with a diagonal measurement of 12 cm. Determine the ratio large screen side : small screen side. Also determine the width and length of the larger screen.



$The\ hypotenuse\ of\ the\ small\ screen\ is\ \sqrt{3^2+5^2} = 5.83cm$

$The\ ratio\ is\ 12{:}5.83.$

$The\ missing\ sides\ can\ be\ found\ with\ the\ ratio\ \frac{5.83}{12} = \frac{3}{a} \rightarrow a = 6.17cm\ and$

$\frac{5.83}{12} = \frac{5}{b} \rightarrow b = 10.3cm.$

2) Determine the missing angle and sides for the pair of similar triangles: $\measuredangle C = 94°\ and\ \measuredangle D\ 68°$

$\measuredangle A = \mathbf{18°} = 180° - 94° - 68°$

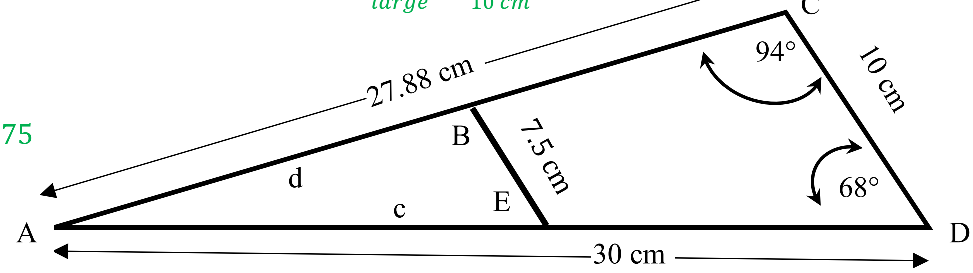
$\measuredangle B = \mathbf{94} = \measuredangle C$

$\measuredangle E = \mathbf{68°} = \measuredangle D$

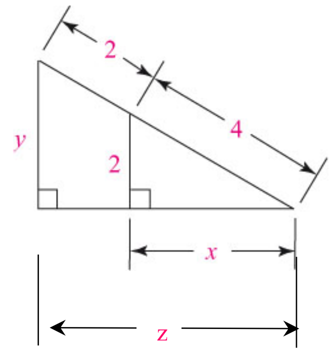
$c = \mathbf{22.5} = 30cm * 0.75$

$d = \mathbf{20.9} = 27.88cm * 0.75$

$\frac{small}{large} = \frac{7.5\ cm}{10\ cm} = 0.75$



3) Determine the missing sides of the similar triangles.

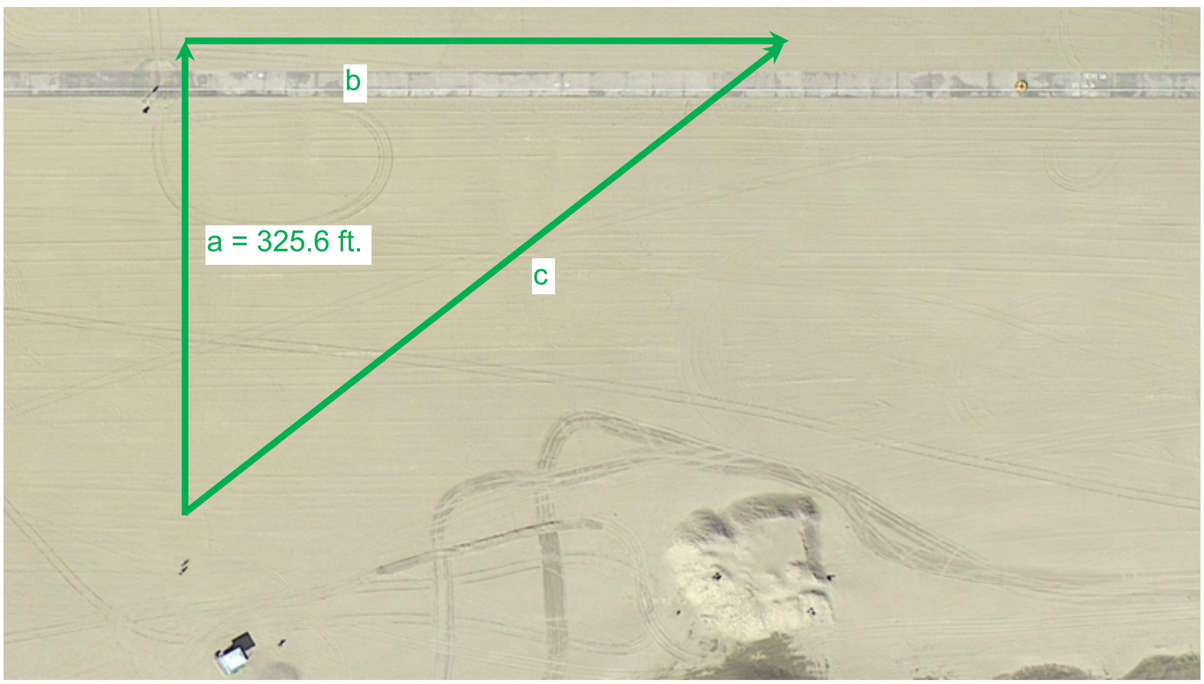


$$\frac{small}{large} = \frac{4}{6} = 0.667 \rightarrow x = \sqrt{4^2 - 2^2}, y = \frac{2}{0.667} \rightarrow z = \frac{3.46}{0.667}$$

x = _3.46____ y = ____3_____ z = ___5.19___

Problem Situation 7.6 – Pulling it all together

4) Where would the taco cart have to be so that both people would reach it at the same time? Draw the point where you think the taco cart should be.



5) Determine the optimum placement of the taco cart and the time it would take Dan and Ben to walk to it.

I typically show them the dimensions and speed again. Watch for too much frustration on this one. Typically there will be at least one person in each group that wants to figure it out.

$First: Write\ down\ what\ I\ know:\quad a = 325.6\ ft$ and $c = \sqrt{a^2 + b^2}$

$$The\ solution\ is\ when\ \frac{a\ ft}{2\ ft/sec} + \frac{b\ ft}{5\ ft/sec} = \frac{c\ ft}{2\ ft/sec} \rightarrow \frac{a}{2}sec + \frac{b}{5}sec = \frac{c}{2}\text{sec}$$

$Second:\ I\ am\ going\ to\ use\ the\ equation\ \frac{a}{2} + \frac{b}{5} = \frac{c}{2}$ and substitute in the value of a and c.

$\frac{325.6}{2}+\frac{b}{5}=\frac{\sqrt{325.6^2+b^2}}{2}$ *then multiply by 2 to simplify the equation.*

$325.6+0.4b=\sqrt{325.6^2+b^2}$ *square both sides.*

$(325.6+0.4b)^2=325.6^2+b^2$ *expand the square.*

$0.16b^2+260.48b+325.6^2=325.6^2+b^2$ *collect like terms*

$b^2-0.16b^2-260.48b+325.6^2-325.6^2=0$

$0.84b^2-260.48b=b(0.84b-260.48)=0$ *solve for b (don't forget the trivial solution)*

$b=0\,ft.\,and\;b=\frac{260.48}{0.84}\,ft=0,310\,ft$

Check my answers: $325.6\,ft\;at\;2\frac{ft}{sec}+310\,ft\;at\frac{5ft}{sec}=225\,sec$

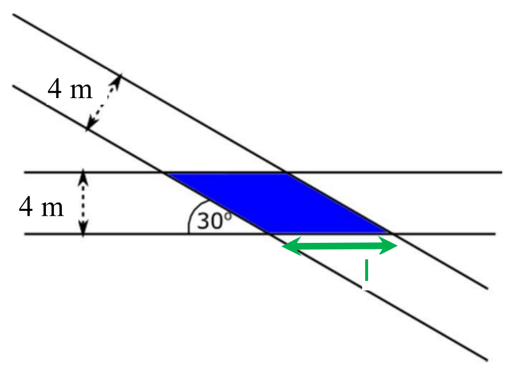
$c=\sqrt{325.6^2+310^2}=449.6\,ft\rightarrow 449.6\,ft\;at\frac{2ft}{sec}=225\,sec$

6) Overlapping roads, each of width 4 meters, are laid across each other at an angle of 30°, as shown in the diagram. Determine the area of the overlap.

To find the area of the shaded parallelogram I need to know the height and the length, A=*h*l.*

h = 4m and I need to determine the length.

A right triangle with one known angle of 30° and the opposite side is 4m. I can use the Sine function to find the hypotenuse which equals the length.

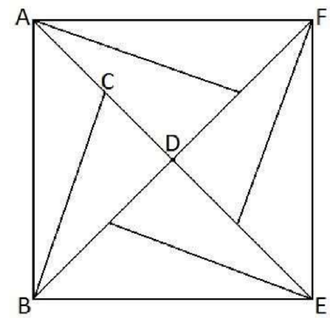


$Sin(30°)=\frac{4m}{l}\quad\rightarrow\quad l=\frac{4m}{Sin(30°)}=8m$

Area = 4m * 8m = 32 m^2

7) The diagram has a rotational symmetry of the order of 4 about D. If $\measuredangle A$BC is 15° and the area of ABEF is 24 cm2, what is the length of CD?

I am looking for the length of CD, which forms one side of a right triangle with BC being the hypotenuse and BD the other side. I can see that $\measuredangle ABD = 45°\ so\ \measuredangle CBD = 45° - 15° = 30°$.



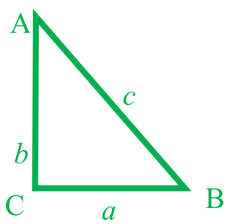
I can find the length of BD because ADB is also a right triangle and I know all the angles and the length of $AB = \sqrt{24cm^2} = 4.9cm.$

$$Sin(45°) = \frac{BD}{HYP} \rightarrow l = \frac{BD}{4.9cm} \rightarrow BD$$
$$= Sin(45°) * 4.9cm = 3.464cm$$

Now I can use the Tan function to determine CD: $Tan(30°) = \frac{CD}{BD}$

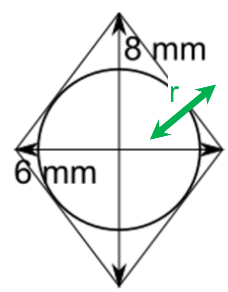
$$Tan(30°) = \frac{CD}{3.464cm} \rightarrow CD = Tan(30°) * 3.464cm = 2cm$$

8) The diagram depicts a diamond ring with the diagonals measuring 6 mm x 8 mm. Determine the radius of the diamond.



There are 4 right triangles with side a = 3mm and side b = 4 mm. This makes the hypotenuse, $c = \sqrt{3mm^2 + 4mm^2} = 5mm$

$Sin(\measuredangle A) = \frac{3mm}{5mm} \rightarrow \measuredangle A = 36.9°$



The radius goes from the center and forms 2 right triangles.

One has the hypotenuse = 4mm and $\measuredangle A = 36.9°$ and r = the opposite side so $r = 36.9° * 4mm = 2.4mm.$