Lesson: Equivalent Fractions
Math Academy – Day 06
Student Notes

Goals:

- Define fractions as numbers
- Define a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts
- Draw fractions as models
- Define a fraction a/b as the quantity formed by a parts of size 1/b
- Interpret a fraction as division of the numerator by the denominator
- Define two fractions as equivalent if they are of the same size
- Recognize equivalent fractions
- Express natural numbers as fractions
- Explain the process of reducing fractions

Prerequisite Knowledge:

- Understand division as a partitioning operation

Activities:

1. Working with a partner, compare your fraction strips. Make sure that you both have all of the same folds and the regions are of the same size for each fraction.

2. Working with a partner, use your fraction strips to sort the fractions $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \frac{1}{10}$ from least to greatest. Note any observations below.

3. Working with a partner, use your fraction strips to sort the fractions $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{2}{3}, \frac{5}{6}, \frac{4}{5}, \frac{9}{10}$ from least to greatest. Note any observations below.

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4. Working with your partner, line up all of your fraction strips. Do you recognize any similar fold lines? Label any fractions that align with each other. Discuss with your partner what this might mean. Be prepared to share your findings with the class.

5. Whole Class Discussion: What did groups find in the previous question?

6. Working with a partner, complete the following questions.
    a. Simplify the following fractions: $\frac{6}{12}$, $\frac{4}{12}$, $\frac{10}{20}$, $\frac{15}{20}$, $\frac{4}{16}$, $\frac{12}{16}$, $\frac{21}{21}$.

    b. Identify which of the following fractions are equivalent: $\frac{12}{30}$, $\frac{24}{40}$, $\frac{3}{9}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{2}{5}$.

7. Working with a partner, on a separate piece of paper, write your names and a fraction that can be simplified (do not write the simplified form of fraction on the paper). Try to be creative. These problems will be posted on the board. (Note: Below on this paper, write the fraction you created along with its simplest form.)

8. Whole Class Discussion: In English, 'to reduce' means to make smaller. Does the same hold true for the mathematical meaning of 'reducing fractions'?

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Instructor Notes

Goals:

- Define fractions as numbers
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Prerequisite Knowledge:

- Understand division as a partitioning operation

Lesson Materials:

- Fraction strips in 3 colors
- Scrap paper for students to write story problems on

Lesson Breakdown:

Activity	Size of Group	Time in Activity Total Time: 55 minutes
Compare fraction strips and their folds	Groups of 2	15 minutes
Sort fraction strips and recognize patterns in the fractions	Groups of 2	10 minutes
Sort fraction strips with different fractions and recognize patterns in the fractions	Groups of 2	10 minutes
Find similar folds and the implications of the fractions that align	Groups of 2	20 minutes
Discuss equivalent fractions and reduction of fractions	Whole class	10 minutes
Simplify fractions and identify equivalence	Groups of 2	10 minutes
Create fractions to be simplified	Groups of 2	10 minutes
Verify the equivalence of the given fractions	Whole class	10 minutes
Discuss reduction as it relates to fractions	Whole class	15 minutes

Activities:

1. Working with a partner, compare your fraction strips. Make sure that you both have all of the same folds and the regions are of the same size for each fraction.

   Students should have the same fold lines. Walk around and discuss the techniques used to make the folds.

2. Working with a partner, use your fraction strips to sort the fractions $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \frac{1}{10}$ from least to greatest. Note any observations below.

   $$\frac{1}{10}, \frac{1}{8}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$$

   Observations may be: (1) as you fold a paper in more equal parts, the parts get smaller (when keeping the numerator the same). (2) Accuracy of the folds are important.

3. Working with a partner, use your fraction strips to sort the fractions $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{2}{3}, \frac{5}{6}, \frac{4}{5}, \frac{9}{10}$ from least to greatest. Note any observations below.

   $$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$$

   Observations may be: (1) as you fold the paper more the regions get smaller. If you always consider all but one piece, then the area considered is larger.

4. Working with your partner, line up all of your fraction strips. Do you recognize any similar fold lines? Label any fractions that align with each other. Discuss with your partner what this might mean. Be prepared to share your findings with the class.

   Similar Fractions:

   One set of similar fractions: $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$

   Another set of similar fractions: : $\frac{1}{3}, \frac{2}{6}$

Another set of similar fractions: : $\frac{2}{3}, \frac{4}{6}$

Another set of similar fractions: : $\frac{2}{3}, \frac{4}{6}$

Another set of similar fractions: : $\frac{1}{5}, \frac{2}{10}$

Another set of similar fractions: : $\frac{2}{5}, \frac{4}{10}$

Another set of similar fractions: : $\frac{3}{5}, \frac{6}{10}$

Another set of similar fractions: : $\frac{4}{5}, \frac{8}{10}$

Another set of similar fractions: : $\frac{1}{4}, \frac{2}{8}$

Another set of similar fractions: : $\frac{3}{4}, \frac{6}{8}$

Another set of similar fractions: : $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{8}{8}, \frac{10}{10}$

Another set of similar fractions: : $\frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \frac{0}{5}, \frac{0}{6}, \frac{0}{8}, \frac{0}{10}$

5. Whole Class Discussion: What did groups find in the previous question?

Previous question, all solutions were listed. Have students create the list of similar fractions. Note which ones have been said. Let students know that "there are more" if they have not exhausted the list. Don't tell them solutions. See if they can keep finding them.

6. Working with a partner, complete the following questions.
   a. Simplify the following fractions: $\frac{6}{12}, \frac{4}{12}, \frac{10}{20}, \frac{15}{20}, \frac{4}{16}, \frac{12}{16}, \frac{21}{21}$.

$$\frac{6}{12} = \frac{1}{2}, \frac{4}{12} = \frac{1}{3}, \frac{10}{20} = \frac{1}{2}, \frac{15}{20} = \frac{3}{4}, \frac{4}{16} = \frac{1}{4}, \frac{12}{16} = \frac{3}{4}, \frac{21}{21} = 1.$$

Students can use extra fraction strip paper if needed to simplify. They can relate the newly created fraction with an already existing fraction strip.

   b. Identify which of the following fractions are equivalent: $\frac{12}{30}, \frac{24}{40}, \frac{3}{9}, \frac{3}{5}, \frac{2}{3}, \frac{2}{5}$.

$$\frac{12}{30} = \frac{2}{5}, \frac{24}{40} = \frac{3}{5}, \frac{3}{9} = \frac{1}{3}, \frac{3}{5}, \frac{2}{3}, \frac{2}{5}$$

Equivalent Fractions are:

1. $\frac{12}{30} = \frac{2}{5}$
2. $\frac{24}{40} = \frac{3}{5}$

7. Working with a partner, on a separate piece of paper, write your names and a fraction that can be simplified (do not write the simplified form of fraction on the paper). Try to be creative. These problems will be posted on the board. (Note: Below on this paper, write the fraction you created along with its simplest form.)

   Walk around and talk to the students about the fractions they are creating. Collect these papers and redistribute them to new groups. Have the groups simplify the fraction they are given. Then, have each group post their fraction and solution on the board. Encourage drawings. Check to see if the students who created the problem and the students who simplified the problem have the same simplified fraction.

8. Whole Class Discussion: In English, 'to reduce' means to make smaller. Does the same hold true for the mathematical meaning of 'reducing fractions'?

   NO!!!! Reducing does not mean to reduce in size. "Reduce" is foul math language. It gives the wrong impression that when we reduce fractions we mean to change the fraction altogether, when this is definitely ***NOT*** the case. Reducing a fraction means to re-write the fraction as an equivalent fraction with the smallest natural numbers for the numerator and denominator ***if possible.***

1. What are equivalent fractions?

2. What does it mean to ‘reduce a fraction'?

3. What does it mean to ‘simplify a fraction’?

4. Which of the following fractions are equivalent fractions: $\frac{5}{6}, \frac{6}{9}, \frac{4}{7}, \frac{1}{3}, \frac{1}{5}, \frac{8}{14}, \frac{15}{18}, \frac{4}{12}, \frac{2}{3}$?

5. Compute $60 - 36 \div 3 \times 4$. Select the appropriate response.
   a. 57
   b. 32
   c. 2
   d. 12

Day 06 Homework Solutions

1. What are equivalent fractions?

   Answer: Equivalent fractions are fractions that appear different when written numerically but represent the same number or quantity. For example, referring to our fraction strips, we can see that $\frac{3}{6}$ and $\frac{1}{2}$ are equivalent fractions because the lines align.

2. What does it mean to 'reduce a fraction'?

   Answer: Reducing a fraction means to re-write the fraction as an equivalent fraction with the smallest natural numbers for the numerator and denominator ***if possible.***

3. What does it mean to 'simplify a fraction'?

   Answer: Simplifying a fraction is the same as reducing a fraction! Simplifying a fraction means to re-write the fraction as an equivalent fraction with the smallest natural numbers for the numerator and denominator ***if possible.***

4. Which of the following fractions are equivalent fractions: $\frac{5}{6}, \frac{6}{9}, \frac{4}{7}, \frac{1}{3}, \frac{1}{5}, \frac{8}{14}, \frac{15}{18}, \frac{4}{12}, \frac{2}{3}$?

   Answers:

   $\frac{5}{6}$ *is equivalent to* $\frac{15}{18}$ OR in math speak: $\frac{5}{6} = \frac{15}{18}$

   $\frac{6}{9}$ *is equivalent to* $\frac{2}{3}$ OR in math speak: $\frac{6}{9} = \frac{2}{3}$

   $\frac{4}{12}$ *is equivalent to* $\frac{1}{3}$ OR in math speak: $\frac{4}{12} = \frac{1}{3}$

   $\frac{8}{14}$ *is equivalent to* $\frac{4}{7}$ OR in math speak: $\frac{8}{14} = \frac{4}{7}$

5. Compute $60 - 36 \div 3 \times 4$. Select the appropriate response.
   a. 57
   b. 32
   c. 2
   d. 12

   Answer: D. The correct order of operations for this problem (division, multiplication, and then subtraction).

$$60 - 36 \div 3 \times 4$$
$$60 - 12 \times 4$$
$$60 - 48$$
$$12$$

   Incorrect Solutions:
   A. Switched the multiplication before division.

$$60 - 36 \div 3 \times 4$$

$$60 - 36 \div 12$$
$$60 - 3$$
$$57 : Incorrect!$$

B. Did the subtraction first, but correctly did division before multiplication.

$$60 - 36 \div 3 \times 4$$
$$24 \div 3 \times 4$$
$$8 \times 4$$

32 : Incorrect!

C. Did the completely reverse order of operations (subtraction, multiplication, then division).

$$60 - 36 \div 3 \times 4$$
$$24 \div 3 \times 4$$
$$24 \div 12$$

2 : Incorrect!