# Introduction

This section is designed to move us through factoring quadratics to quadratic applications. The first activity focuses on the use of factors, the second on applications of quadratics once the students have mastered factoring strategies. All activities can be done as a classroom discussion in small groups, remotely in breakout rooms, or in an online discussion board.

# ACTIVITY FOR CONNECTION: FINDING FACTORS

## The Question:

Suppose you want to tile a floor measuring 12 x 15 feet and you want to use only whole tiles which are all the same size. Tiles are available in squares which are 4, 5, 6, 8, 9, 12 inches a side. Which size tiles can you use without cutting tiles? What if the floor measured 6 x 10? [[1]](#footnote-1)

## Materials list:

### For Each Student

* Perfect Fit handout (see Appendix)

## The Process:

1. Start with noting that the floor and tiles are given in different units of measure. Convert the floor measurements from feet to inches.

1. Check each tile length to see which, laid end to end, would cover the floor in both directions with integer multiples (no partial tiles so no cutting).

, , , ,

, , , ,

Possible tile lengths: 4, 6, 9, 12 inches

For the floor measuring 6 x 10 feet or 72 x 120 inches,

, ,

, , , ,

Possible tile lengths are 4, 6, and 8 inches

**Discussion Questions**

What are other times when knowledge of factors comes in handy?

* Dividing a pan of brownies evenly among your classmates,
* making change – a dollar can be divided into 100 pennies: 1x100, 20 nickels: 5 x 20, 10 dimes: 10 x 10, 4 quarters 4x25,
* understanding time – every hour divides into four 15-minute intervals, six 10-minute intervals, twelve 5-minute intervals (used for taking pulse rates without having to count for 60 minutes)

# EXTENDING THE LESSON

You can use any of the questions in this section to continue the discussion or use them as group work.

These can be used similarly for a remote class, using breakout rooms for discussion and comparing results.

For a completely online class, assign the problem handout and ask the students to submit their work individually.

## Equal Squares

Cut a piece of paper with a base of 18 cm, and a height of 27 cm into identical squares with the largest possible dimensions. How many squares can be made without having any remaining paper?

Find the factors of each dimension

, ,

,

The largest factor that each dimension has in common is 9 so cut the paper into 9cm squares.

## Product Consistency

A bakery wants to introduce fruit pies to its product list. They want to make some samples out of 48 blueberries, 24 raspberries, and 36 slices of kiwi, and they want to use all the fruit. How many sample pies can be made so that the same amount of each type of fruit is in each pie?

Find the factors of each fruit count

, , , ,

, , ,

, , , ,

The answer depends on how large the sample pies will be and how much fruit per bite the bakery wants their patrons to experience.

* Two pies will have 24 blueberries, 12 raspberries and 18 slices of kiwi;
* Three pies will have 16 blueberries, 8 raspberries and 12 slices of kiwi;
* Four pies will have 12 blueberries, 6 raspberries and 9 slices of kiwi;
* Six pies will have 8 blueberries, 4 raspberries and 6 slices of kiwi;
* Twelve pies will have 4 blueberries, 2 raspberries and 3 slices of kiwi.

## Emergency Relief

A local community center is packing backpacks for children returning to school after a recent Hurricane. They wanted to divide 144 pencils, 90 note pads, and 36 granola bars evenly among the backpacks, without any supplies remaining. Find the largest number of backpacks they can pack.

Find the factors of each item count.

, , , , , ,

, , , ,

, , , ,

The largest factor that each dimension has in common is 18 so the community center can pack 18 backpacks with 8 pencils, 5 notebooks, and 2 granola bars.

# Numbers Games

The appendix offers several puzzles for warmups, group work, or just for fun. If working with small groups, assign a different puzzle to each group. Groups can share answers or, to save time, distribute the solution set after the exercise is done. The puzzles may be different, but all use factors, providing a segway to the topic of factoring polynomials. [[2]](#footnote-2)

## Solutions

Multiplication Squares

A.

|  |  |  |
| --- | --- | --- |
| 4 | 5 | 7 |
| 2 | 3 | 9 |
| 6 | 8 | 1 |

B.

|  |  |  |
| --- | --- | --- |
| 3 | 5 | 9 |
| 4 | 7 | 2 |
| 8 | 1 | 6 |

C.

|  |  |  |
| --- | --- | --- |
| 8 | 2 | 6 |
| 5 | 7 | 4 |
| 1 | 3 | 3 |

Magic Multiplication Squares

|  |  |  |
| --- | --- | --- |
| 12 | 1 | 18 |
| 9 | 6 | 4 |
| 2 | 36 | 3 |

|  |  |  |
| --- | --- | --- |
| 28 | 1 | 98 |
| 49 | 14 | 4 |
| 2 | 196 | 7 |

|  |  |  |
| --- | --- | --- |
| 75 | 1 | 45 |
| 9 | 15 | 25 |
| 5 | 225 | 3 |

Shared Factors

Large and Small Factors

Twelve Factors

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 10 | 12 | 15 | 20 | 30 | 60 |
| 1 | 2 | 3 | 4 | 6 | 8 | 9 | 12 | 18 | 24 | 36 | 72 |
| 1 | 2 | 3 | 4 | 6 | 7 | 12 | 14 | 21 | 28 | 42 | 84 |
| 1 | 2 | 3 | 4 | 6 | 9 | 10 | 15 | 18 | 30 | 45 | 90 |
| 1 | 2 | 3 | 4 | 6 | 8 | 12 | 16 | 24 | 32 | 48 | 96 |

# Factoring Challenges

Use these questions while teaching the factoring strategies. While these are not problems in “context” I often find that some students finish work before other groups and are up for more challenges. Below are questions to provoke some thought outside of the algorithmic process of factoring.

## Factoring by grouping

Factor: by grouping[[3]](#footnote-3)

Reorder and group:

Factor GCF from each group:

Factor out the common binomal:

## Factoring the difference of perfect squares

Show how you can use the difference of two squares to find the products of 4238 and 876. [[4]](#footnote-4)

Rewrite as the product of conjugates, multiply the binomials and add the two terms.

Perhaps show a segment of the Ted Talk

<https://www.ted.com/talks/arthur_benjamin_a_performance_of_mathemagic>

## Factoring sum and difference of cubes; difference of squares

Factor as the difference of two squares. Then factor the same polynomial as the difference of two cubes. Do the results agree? Why or why not? [[5]](#footnote-5)

Rewrite as the difference of cubes and factor:

Factor the difference of squares

Rewrite as the difference of squares and factor:

Factor the sum and difference of cubes

The results don’t look the same but they are equivalent because

## Solving Quadratics

The length of a rectangle is 2 feet greater than the width, and the area is 18 square feet. Find the width of the rectangle. [[6]](#footnote-6)

Set up the equation. Can you solve it? Why not?

#### Identifying the variables

* is the width of the rectangle
* is the length of the rectangle

#### Setting Up

#### Solving

#### Final Answer

Cannot be solved by factoring. There are no factors of that sum to .

# ACTIVITY FOR CONNECTION: QUADRATIC APPLICATIONS

The construction of the Golden Gate Bridge was completed in 1937. It was the longest suspension bridge until 1981 and the tallest of any type of bridge until 1993. Built to withstand strong currents, blinding fog, high winds up to 90mph, and earthquakes measuring up to 8.0 on the Richter Scale, the bridge was named one of the seven civil engineering wonders of the United States by the American Society of Civil Engineers in 1994. [[7]](#footnote-7)

## The Question:

Determine how far would you walk from one end of the suspension span to the other? While you are walking, would you be able to touch the Main Span cable?

## The Process:

The bridge is made up of the main center span and two side spans, the distance of each will have to be calculated separately.

## Materials list:

### For Each Student

* Golden Gate Bridge Facts handout (see Appendix)

## The algebra behind the activity

### The Main Span

To find the length of the span (distance between the two towers) a parabola represented by the equation  is superimposed over a picture of the Golden Gate Bridge [[8]](#footnote-8) where is the distance of the main cable in feet from the roadway and is the distance of the cable in feet from the South Tower (San Francisco side). Points are labelled for reference.

*x*

*y*

A picture containing water, outdoor, sunset, shore

Description automatically generated

(b,a)

(0,a)

(,c)

Use the coordinate grid and what you know of finding points on a graph to find the distance between the two spans.

1. Find the height of the South Tower by calculating the value for  when
2. The two towers are the same height so use the height found in a to find when .

is the South Tower; is the North Tower (Marin side).

The distance between the two Towers, the Main Span, is 4200 feet.

### The Side Spans

The two ends of the bridge each form a right triangle. If the length of the cable from the top of the tower to the base at the end of the bridge is 1231 ft. Use Pythagorean’s theorem to find the distance between the towers and the end of the bridge.

A picture containing water, outdoor, sunset, shore

Description automatically generated

d

d

1231 ft ft

1231 ft

Distance:

The total walking distance from one end of the bridge to the other is the main span plus the north and south side spans.

Total walking distance =

To convert to miles:

To determine if you can touch the Main Span cable, calculate the height of the cable at its lowest point of the main span. Parabolas are symmetrical, so the lowest point will be halfway between the two towers, that is, the point labeled on the first picture.

when

A tall person that can jump high enough might be able to reach the Main Span cable.

## Discussion Questions

Do you have a favorite suspension bridge? What elements of nature had to be overcome in building the bridge? (This could be a project assignment)

To discuss the enormity of a project such as building a bridge and the impact to society and economy, look at the video embedded in the article found at <https://www.history.com/topics/landmarks/golden-gate-bridge>

# Extending the Lesson: Unsafe Sidewalks

## Introduction:

Sticking with the theme of infrastructure, we can look at heaves in the roadbed, braking distance, and blind spots. As with the previous activity, any of these questions can be used to continue the class discussion or for individual or group work.

## Sidewalk Cracks

The most common reasons for cracking in sidewalks are the expansion and contraction in hot and cold temperatures, erosion caused by poor drainage, and unstable soil conditions under the slab.

When concrete expands, it pushes against anything in its way (a brick wall or adjacent slab for example). When neither has the ability to flex, the expanding pressure can be enough to cause concrete to crack. Expansion joints are used to put space between two solid surfaces.

**[](http://epod.typepad.com/.a/6a0105371bb32c970b0120a5633e90970c-pi)**Why is this sidewalk buckled? All materials expand and contract with daily and seasonal temperature variations. Solid rock, or man-made substances such as concrete, or even steel, can generate huge internal forces when there is not enough room to expand. The photo [[9]](#footnote-9) was taken on a hot (near 100 F° or 37.4 C° summer day (July 13, 2003) in Great Falls, Montana, and the concrete sidewalk rafted as a result of the high temperatures. However, even though expansion cracks are evident, they were not sufficient to handle the pressures of the constrained concrete. [[10]](#footnote-10)

## The Question:

If you worked for the Department of Public Works in your town or this happened in front of your house, you would need to repair the sidewalk before the neighborhood kids skating in front of your house take a headlong dive. You’ve measured the slabs in front of your house and found they are each 3 feet long; they heaved vertically 4 ¼ inches. Calculate the width of the expansion cracks needed between the slabs to prevent this from happening again.

## The Process:

1. Convert numbers to be used in calculating the length of the diagonal to the same units.

4 ¼ in

3ft

D

1. Calculate the diagonal length resulting from the heave.
2. Find the amount of spacing needed to prevent this type of cracking again.

## Materials list:

### For Each Student

* Road Safety handout (see Appendix)

## The algebra behind the activity

### Identifying the variables

* *D* is the length of the diagonal

### Setting up the problem

1. Using the Pythagorean Theorem: where is the length of the slab, is the height of the heave and is the length of the diagonal.

## Conclusion:

The original slabs were 36 inches, the diagonal of the heave was 36.25 inches. The spacing will need to be a total of inches (there are two sides to the heave), or ½ of an inch.

# Practice Exercises

## Braking Distance

The braking distance in feet required to stop a car traveling at miles per hour on dry level pavement can be approximated by . If the braking distance is 44 feet, calculate the speed of the car.[[11]](#footnote-11)

Solution:

and

and

### Identifying the variables

* is the braking distance
* is the vehicle speed on dry level pavement

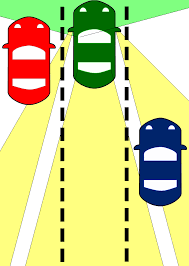
### Setting Up the problem

Find when so

### Final Answer

The vehicle is moving 22 mph (speed can not be negative)

## Blind Spots



All cars have a blind spot where it is difficult for the driver to see a car behind and to the right. The area of the rectangular blind spot shown is 54 ft2. Its length is 3 feet longer than its width. Find its dimensions. [[12]](#footnote-12)

### Identifying the variables

* is the width of the blind spot
* is the length of the blind spot

Solution

and

and

### Setting Up the problem

so

### Final Answer

Distance is positive so the width is and the length is

Firework Shows

Organizers of firework shows use of quadratic and linear equations to help them design their programs and provide for the safety of the audience. Shells contain the chemicals that produce the bursts we see in the sky. At a firework show, the shells are shot from mortars and when the chemicals inside the shells ignite, they explode, producing the brilliant bursts we see in the night sky.

At a fireworks show, a 3-inch shell is shot from a mortar at an angle of 75°. The height, *y*, (in feet), of the shell *t* seconds after being shot from the mortar is given by the quadratic equation:

And the horizontal distance of the shell from the mortar, *x* (in feet), is given by the linear equation:

1. How high is the shell after 3 seconds?
2. What is the shell’s horizontal distance from the mortar after 3 seconds?
3. The shells are designed to explode at their maximum height in flight. How high is the shell when it bursts after 4.5 seconds?
4. What is the shell’s horizontal distance from its launching point when it explodes?
5. If the shell doesn’t explode, what is the shell’s horizontal distance from its launching point when it hits the ground? [[13]](#footnote-13)

### Identifying the variables

* is the horizontal distance of the shell from the launching point at time,
* is the height of the shell at time,
* is time in seconds

### Setting Up the problem

3. First solve for the number of seconds the shell is in flight; then substitute 9 for

### Final Answer

2. ,

## Endangered Species

The crocodile, an endangered species, is the subject of a protection program. The mathematical model  descries the crocodile population after *t* years of the protection program.

1. How long will the program have to be continued to bring the population up to 7250?
2. The graph of the crocodile population as a function of time is shown here. [[14]](#footnote-14) Identify the ordered pair on the function’s graph corresponding to your solution in part (a).
3. Interpret the meaning of the solution? [[15]](#footnote-15)

### Identifying the variables

Solution

and

and

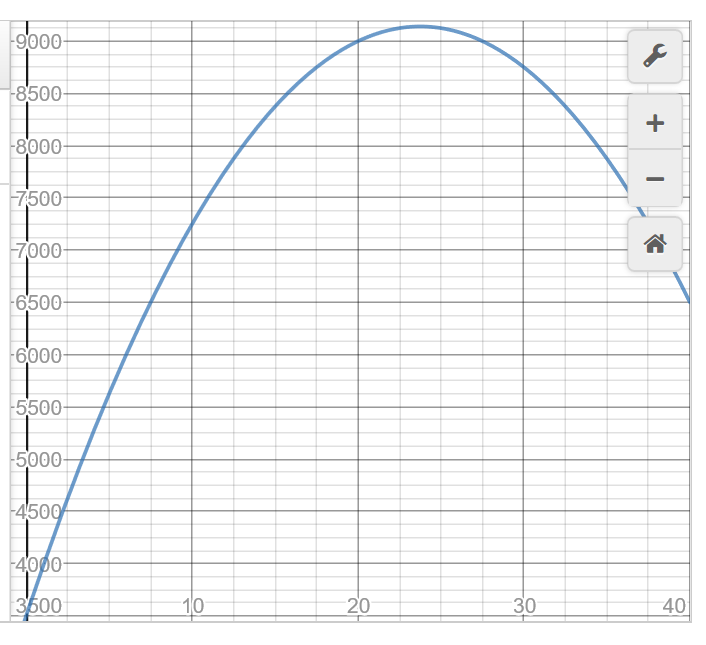
* is the crocodile population after years
* is time in years

### Setting Up the problem



### Final Answer

1. while the population is increasing, 37.5 years when the population is decreasing



1. There are two points. In the early years the population is decreasing so after 10 years the population is 7250. At around 22 years the population starts to decrease so after 37.2 years the population is again 7250.

## Road Speed

Safety research uses the model  to estimate the shortest distance in feet, , in which a car can be stopped at various speeds, (miles per hour). If it took a car 550 feet to stop, estimate the car’s speed at the moment the brakes were applied.[[16]](#footnote-16)

### Identifying the variables

Solution

and

and

* is the shortest stopping distance of a vehicle
* is the speed of the vehicle

### Setting Up the problem

### Final Answer

mph

## Emergency Relief

At times when emergency supplies are needed in an area whose terrain doesn’t support a landing strip, supplies must be dropped. How long will it take a package of medical supplies to reach the ground if it is released from a plane at an altitude of 1600 ft.? The initial velocity is 0 ft/sec so that the distance, *d*, of the object from the ground *t* seconds after it is released is given by  [[17]](#footnote-17)

### Identifying the variables

Solution

and

and

* is the distance from the ground
* is time in seconds

### Setting Up the problem

### Final Answer

seconds

## Air Pollution

The amount of particulate pollution in the air depends on the wind speed , among other things, with the relationship between and approximated by , where is in ounces per cubic yard and is in miles per hour. Interpret the and intercepts. [[18]](#footnote-18)

Solution

and

and

### Identifying the variables

* is the ounces of particulate pollution per cubic yard
* is wind speed in miles per hour

### Setting Up the problem

The intercept is when :

The intercept is when :

### Final Answer

-intercept: ; there are 25 ounces of particulate pollution per cubic yard when there is no wind

-intercept: ; there is no particulate pollution when the wind speed is 50mph

## Drug Sensitivity

Chart, line chart

Description automatically generatedThe sensitivity to a drug is related to the dosage size, , is represented by .

Find when . Interpret the solution. Since this equation is a parabola (and parabola are symmetrical) can you find the dosage that give the maximum effect. [[19]](#footnote-19)

### Identifying the variables

* is the dosage size
* is the sensitivity to a drug

### Setting Up the problem

For:

Solution

and

and 0

The graph of the equation is a parabola, parabola’s are symmetrical so the maximum point (vertex) will be half way point between and

### Final Answer

: no drug was administered

: There will be no sensitivity to the drug with a dosage of 1000 or more. The maximum effect would be a dose of 500.

## Adventure Course



A builder of a high ropes adventure course wants to include a rope ladder whose length is 8 meters longer than the horizontal distance between the base of the ladder and the top of the tower. The vertical height is one meter less than the ladder itself. How high is the top of the ladder?

### Identifying the variables

Solution

and

and

* is the horizontal distance between base and ladder
* is the length of rope ladder
* is the vertical height

### Setting Up the problem

### Final Answer

13 meters ( so the ladder is )

## 

## Architecture

A picture containing sky, outdoor, nature, rainbow

Description automatically generatedThe shape of the famous “Gateway to the West” arch in Saint Louis (Shown on the right [[20]](#footnote-20)) can be modeled by a parabola. The equation for one such parabola is: where x and y are in feet. Approximately how far do you have to walk to get from one side of the arch to the other? [[21]](#footnote-21)

### Identifying the variables

* is the distance from the left base
* is the height of the arch

Solution

and

and

### Setting Up the problem

### Final Answer

# Article and Video Links

## Suspension bridge collapse

<https://www.youtube.com/watch?v=XggxeuFDaDU>

<https://www.youtube.com/watch?v=j-zczJXSxnw>L

<https://www.youtube.com/watch?v=KRtefycRGdE>

## Factsheets for the Golden Gate Bridge:

<https://www.fhwa.dot.gov/candc/factsheets/goldengatebridge.pdf>

https://www.goldengate.org/bridge/history-research/statistics-data/design-construction-stats/

## Quadratic Functions in Real World

<https://youtu.be/He42k1xRpbQ>

# Appendix – Student Handouts

* Perfect Fit and Extending the Lesson
* Number Games
* Golden Gate Bridge
* Unsafe Sidewalk
* Practice Exercises

1. Stanley Gudder. A Matheatical Journey. McGraw-Hill Publishing Company. 1976 (pg 121). [↑](#footnote-ref-1)
2. www.kenkenpuzzle.com/game [↑](#footnote-ref-2)
3. Alan S. Tussy, R. David Gustafson. Elementary Algebra. Thomson Brooks/Cole. 2008. (pg 482) [↑](#footnote-ref-3)
4. Richard N. Aufmann, Joanne S. Lockwood. Beginning & Intermediate Algebra. Cengage Learning. 2013. [↑](#footnote-ref-4)
5. Rosemary M Karr, Marilyn B Massey, R. David Gustafson. Beginning Algebra: A Guided Approach. Cengage Learning. 2015. (pg 337) [↑](#footnote-ref-5)
6. Rosemary M Karr, Marilyn B Massey, R. David Gusafson. Beginning Algebra: A Guided Approach.Cengage Learning. 2015. (pg 350) [↑](#footnote-ref-6)
7. <https://www.history.com/topics/landmarks/golden-gate-bridge> [↑](#footnote-ref-7)
8. https://upload.wikimedia.org/wikipedia/commons/2/2a/Golden\_Gate\_Bridge\_Dec\_15\_2015\_by\_D\_Ramey\_Logan.jpg [↑](#footnote-ref-8)
9. Universities Space Research Association - Earth Science Picture of the Day <https://epod.usra.edu/blog/2003/08/expansion-cracks.html> [↑](#footnote-ref-9)
10. Provided by: [Kent Barnes](mailto:kent_barnes@msn.com), [Montana Department of Transportation](http://www.mdt.state.mt.us/), Summary authors & editors: [Jim Foster](mailto:james.foster@nasa.gov); [Kent Barnes](mailto:kent_barnes@msn.com) [↑](#footnote-ref-10)
11. Modified from: Gary K. Rockswold and Terry A. Krieger. Beginning & Intermediate Algebra. Pearson Education, Inc. 2013. pg 400. [↑](#footnote-ref-11)
12. Modified from: Alan S. Tussy, R. David Gustafson. Elementary Algebra. Thomson Brooks/Cole. 2008. (pg 550) [↑](#footnote-ref-12)
13. Sherri Messersmith, Robert S. Feldman. Beginning & Intermediate Algebra. McGraw Hill 2016. (pg 497) [↑](#footnote-ref-13)
14. <https://www.graphcalc.com/online-graphing-calculator/> [↑](#footnote-ref-14)
15. Robert Blitzer. Introductory Algebra for College Students. Prentice – Hall, Inc. 1998. Pg 549. [↑](#footnote-ref-15)
16. Robert Blitzer. Introductory Algebra for College Students. Prentice – Hall, Inc. 1998. Pg 719 [↑](#footnote-ref-16)
17. Modified from: Richard N. Aufmann, Joanne S. Lockwood. Beginning & Intermediate Algebra. Cengage Learning. 2013.pg 401. [↑](#footnote-ref-17)
18. Harshbarger, Ronald J. and Lisa S. Yocco. College Algebra in Context, 4th Edition. Pearson. 2013. pg 181. [↑](#footnote-ref-18)
19. Modified from: Harshbarger, Ronald J. and Lisa S. Yocco. College Algebra in Context, 4th Edition. Pearson. 2013. pg 181. [↑](#footnote-ref-19)
20. <https://commons.wikimedia.org/wiki/File:Gateway_Arch,_St._Louis,_Missouri.jpg> [↑](#footnote-ref-20)
21. McKaegue, Charles P. Intermediate Algebra, eighth edition. Thomson Brooks/Cole. 2008. pg. 503. [↑](#footnote-ref-21)