# Introduction

There are two different activities presented. The first to be used as the unit on Radical Expressions gets introduced and the second at the end of the unit when students have learned to solve equations with radicals.

# Activity for Connection: Polution Plume

This activity is for Evaluating Radicals.

## The Question

If our rivers and oceans are to be preserved for future generations, we need to work to eliminate pollution from our waters. If a river is flowing at 1 meter per second and a pollutant is entering the river at a constant rate, the shape of the pollution plume can often be modeled by the simple equation where is the width of the plume and is the distance from the source (both in meters).

A long straight river, 100 meters wide, is flowing at 1 meter per second. A pollutant is entering the river at a constant rate from one of its banks. As the pollutant disperses in the water, it forms a plume that is modeled by the equation , where x is the distance from the source in meters and y is the width of the plume in meters. How far down river from the source of the pollution does the plume reach the other side of the river? [[1]](#footnote-1)



Photograph by Kaentian Street

## River Pollution

Waste filled water is dumped into a river, polluting it for the people and animals who use it as a source for eating and drinking. [[2]](#footnote-2)

|  |  |
| --- | --- |
| *x* |  |
| 25 |  |
| 49 |  |
| 100 |  |

**The process:**

1. Complete the table.

Chart, line chart

Description automatically generated

1. Graph the results to visualize the pollutant plume. Imagine that the river is flowing from left to right, parallel to the x-axis, with the x-axis as one of its banks. The pollutant is entering the river from the bank at (0,0)
2. How far down river from the source of the pollution does the plume reach halfway across the river?
3. How far down river from the source of the pollution does the plume reach the other side of the river?

## Materials list:

### For Each Student

* Pollution Plume handout page 1

## Discussion Questions

<https://www.sciencefriday.com/videos/breakthrough-bitter-water/>

## The algebra behind the activity

### Identifying the variables

In the equation, .

* *x* is the distance from the source of the pollutants
* *y* is the width of the pollutant plume

|  |  |
| --- | --- |
| *x* |  |
| 25 | 5 |
| 49 | 7 |
| 100 | 10 |

### Setting up the Problem

1. Complete the table .
2. Graph the results.

Chart, line chart

Description automatically generated If this is done at the start of the unit on radicals, students may have to guess and check based on values for ; some students may already know to square the value for to get a value for .

1. Given the river is 100 meters wide, halfway across would be 50 meters. .

# Extending the Lesson: Velocity vs. Height

## Introduction:

The square root is related to one of the most revolutionary experiments in the history of science. Working some 500 years ago, Galileo Galilei discovered that contrary to ancient authority, the speed of falling objects, which he had dropped from an Italian cathedral tower, was proportional to the square root of the height from which they were dropped. Because of such experiments, Galileo is considered to be the founder of modern physics. [[3]](#footnote-3) [[4]](#footnote-4)



photograph by Nancy Fleming

## The Problem:

The expression can be used to determine the velocity of a free-falling object in feet per second, where ft/sec2, and the distance the object has fallen is *h* ft. If a ball has fallen 60 ft, is the ball’s velocity double that of a ball that has fallen 30 ft? Explain your reasoning.

## Materials list:

### For Each Student

* Velocity vs. Height handout page

## The algebra behind the activity

The height has doubled between 60 and 30 ft but the relationship between velocity and height is not linear.

The student might compare the velocity from a height of 30ft and at 60ft:

Or the student might note the velocity at 60 ft,

## Conclusion:

The change in velocity is not increased at the same factor but by the square root of the factor.

# Practice Exercises

## Water Supply

Evaporation occurs when the sun’s heat causes the lake’s water molecules to gain enough energy to escape from the lake. The amount of evaporation, in inches per day, of a large body of water can be modeled by the formula  where is the surface area of the water in square miles, is the average wind speed of the air over the water in miles per hour, and is evaporation in inches per day. Determine the evaporation on a lake whose surface area is 9 square miles on a day when the wind speed over the water is 10 miles per hour. [[5]](#footnote-5) [[6]](#footnote-6)

Solution

Evaporation can become a key concern when working to provide clean water solutions to developing communities. Just when will capacity from other water sources need to be increased because of the evaporation of a local water source.

<https://www.actionagainsthunger.org/story/no-water-and-no-harvest-somalia-drought>

### Identifying the variables

Over a surface area of 9 squared miles is 3,484,800 cubic feet:

Changing to gallons

If an individual needs 5 gallons of water per day, that’s enough for 5,213,260 people.

* is evaporation rate in inches per day
* is the surface area of the lake in square miles
* is the wind speed in mph

### Setting Up the Problem

### Final Answer

The evaporation rate is 1/6 of an inch per day.

## Vehicle Speed

Police use the model to estimate the speed of a car (, in miles per hour) with the length (, in feet) of its skid marks on dry pavement. What is a reasonable estimate for a car’s speed if skid marks measure 225 feet? If the posted speed limit is 50 miles per hour and the motorist tells the officer he was not speeding, should the officer believe him?[[7]](#footnote-7)

### Identifying the variables

* is the speed of the car in mph
* is the length of the skid marks in feet

### Setting Up the Problem

### Final Answer

Solution

The estimated speed of the car was 73.5 mph.

## Road Safety - Bikes

What is the maximum velocity that a racing cyclist can turn a corner without tipping over? The answer is provided by the mathematical model where is the maximum velocity in miles per hours and is the radius of the corner, in feet. What is the maximum velocity that a cyclist should travel around a corner of radius 9 feet without tipping over?[[8]](#footnote-8)

### Identifying the variables

* is the speed of the cyclist
* is the radius of the corner

### Setting Up the Problem

Solution

### Final Answer

The cyclist should not exceed 12 mph to navigate the corner safely.

## **Water through a Fire Hose**

The rate at which water flows through a particular fire hose, *R*, in gallons per minute, can be approximated by the formula , where *d* is the diameter of the nozzle, in inches, and *P* is the nozzle pressure, in pounds per square inch. If a nozzle has a diameter of 2.5 inches and the nozzle pressure is 80 pounds per square inch, find the flow rate. [[9]](#footnote-9)

### Identifying the variables

Solution

* is the number of gallons of water per minute
* is the nozzle pressure
* is the diameter of the nozzle

### Setting Up the Problem

### Final Answer

The rate of water flow is approximately 1565 gallons per minute.

## Windmills

The power generated by a windmill is related to the speed of the wind by the formula

, where *S* is the speed of the wind in miles per hour and *P* is the power in watts. Find the speed of the wind when the windmill is producing 80 watts of power. [[10]](#footnote-10)

### Identifying the variables

Solution

* is the speed of the wind
* is the power generated

### Setting Up the Problem

### Final Answer

The wind speed is approximately 15.9 *mph* .

## Road Safety - Cars

The maximum velocity (, in miles per hour) that an automobile can travel around a curve with a radius of feet without skidding is described by the  If the curve has a radius of 250 feet, find the maximum velocity a car can travel around it without skidding. [[11]](#footnote-11)

### Identifying the variables

Solution

* is the speed of the car
* is the radius of the curve

### Setting Up the Problem

### Final Answer

The maximum speed (velocity) a car can travel around the curve is 25 *mph*.

## Infant Mortality Rate

The U.S. infant mortality rate has been declining steadily. The infant mortality rate, , defined as deaths per 1000 live births, can be estimated by the function  where *t* is years since 1969 and . Find the infant mortality rate in 1970 and 1996. [[12]](#footnote-12)

What is the significance of the interval given for time?

<https://www.macrotrends.net/countries/USA/united-states/infant-mortality-rate>

### Identifying the variables

* is the infant mortality rate
* is the number of years since 1969; is the year 1969; is the year 1975

Solution

### Setting Up the Problem

### Final Answer

The infant mortality rate in 1969 was approximately 29 deaths for every 1000 live births. In 1975 the mortality rate dropped to approximately 3 deaths for every 1000 live births.

## Growing Bacteria

The function  approximate the number of bacteria in a certain culture after *t* hours. The initial number of bacteria is determined when . What is the initial number of bacteria? How many bacteria are there after ½ hour. [[13]](#footnote-13)

### Identifying the variables

* is the number of bacteria in a culture
* is the hours the bacteria has been growing

Solution

### Setting Up the Problem

### Final Answer

The culture started with 1024 bacteria; after ½ hour there were 1448.

# Activity for Connection: Water Storage

This activity uses system of equations, conversion (or unit analysis), solving equations with radicals, rationals, and higher order polynomials (cube root property). It may work best used as the last exercise of the term.

## The Question

[](https://www.iom.int/sites/default/files/migrated_files/pbn/ss20120703_1.jpg)The IOM (International Organization for Migration) is one organization responsible for water and sanitation services to refugees. Initially, IOM delivers water by truck to outlying areas of the camp to reduce the walk for households to access safe water. In a second phase, they dig boreholes, install pumps, and build water storage, bringing a source of water to the camp without trucking. Find the dimensions of the water storage containers.

Formulas needed:

where

* *r* is radius,
* *V* is volume,
* *h* is height

## The process:

The daily water requirement for each of 27,000 refugees is 20 liters per day. How much water needs to be provided daily?

1. If the recommendation is to have a 3-day supply stored at any given time, what must be the combined capacity of the tanks?
2. There will be a total of twelve tanks distributed throughout the camp whose capacity will be measured in cubic meters. What is the capacity of each tank?
3. The radius of each tank is to be the height of the tank. What are the dimensions of the tank?



If you are interested in just how much more math goes into completing this project, see “*Solar Pumping Basics.”*

<http://documents1.worldbank.org/curated/en/880931517231654485/pdf/123018-WP-P159391-PUBLIC.pdf>

## Materials list:

### For Each Student

* Water Storage handout

## Discussion Questions

What math concepts did you have to dredge up to complete this task?

## The algebra behind the activity

### Identifying the variables

The formula for the volume of a cylinder is  so that in terms of the radius,

* *V* is the volume of the storage tank
* is the height of the tank
* is the radius of the tank

**Setting up the Problem**

1. .
2. First convert liters to square meters for total capacity needed:

There will be 12 tanks so each tank must be or 135 cubic meters.

1. Since the radius will be 2/3 the height

and we will solve

### Solving the Problem

There are two equations, using the substitution method of solving systems we start with

and substitute the value for fond in step 3.

Now solve the radical equation:

### Conclusion

The height of each take will be 3.6 meters. The radius will be or 2.4 meters.

# Extending the Lesson

**Introduction:**

When a tsunami approaches a shoreline, the depth of the water decreases, and the tsunami slows down. As tsunami waves slow down, they move closer together and rise in height. The energy, height, and speed of the tsunami results in a destructive path.

The Pacific Tsunami Warning Center is responsible for monitoring earthquakes that could potentially cause tsunamis in the Pacific Ocean. By measuring the water level and calculating the speed, the arrival of a tsunami to a populated coastline can be predicted. The speed is given by the equation  where *s* is the speed in meters per second, *g* is gravity (9.8 m/sec2), *d* is the depth of the ocean in meters. The average depth of the Pacific Ocean, in open ocean, is 4280 meters.

1. What is the speed of a tsunami in open ocean?

The epicenter of 2011 Tohoku earthquake and subsequent tsunami was 72 km east of the Oshika Pennisula, the closest land point. With tsunami waves travelling about 700 km/hour, Sendai residents had an 8 – 10 minute warning.

1. At this speed, estimate the depth of the ocean.

## The algebra behind the activity

### Identifying the variables

In the equation,

* *s* is the estimated speed of the tsunami in meters per second
* *g* is gravity given at
* is the depth of the ocean in meters

### Setting up the Problem

1. Substitute the given depth of the ocean and the constant for gravity

Solution

1. Units of measure are not consistent so either the speed of the tsunami must be converted to m/sec or the gravity constant must be converted to km/hour.

Substitute the given depth of the ocean and the constant for gravity

**Conclusion**

The speed of a tsunami in open ocean is 737 km/hr. The estimate depth of the ocean when the tsunami speed was 700 km/hr is 3858 meters.

For more information visit:

<https://en.wikipedia.org/wiki/2011_T%C5%8Dhoku_earthquake_and_tsunami>

# Practice Exercises – Solving Equations with Radicals

## Vehicle Speed

Police use the formula  to estimate the speed at which a car traveled at the time of an accident, where is the length in feet of the skid marks made by the car in the accident. A police car traveling 30 miles per hour simulates the conditions of the accident. In the model, *p* is the length of the skid marks made by the police test car. A car traveling at 90 miles per hour was in an accident that was simulated by a test car. If the length of the skid marks left by the police test car was 100 feet, what was the length of the skid marks left at the time of the accident? [[14]](#footnote-14)

note the development of the formula as our ability to deal with more complex radicals increased.

Solution

### Identifying the variables

* is the speed of the car in mph
* is the length of the skid marks in feet
* is the length of the skid marks made by the policy test car

### Setting Up the Problem

### Final Answer

The car would have made skip marks of 900 feet.

Highway Design

A highway curve banked at 8° will accommodate traffic traveling at speed *(mph*) if the radius of the curve is feet, according to the equation . If highway engineers expect traffic to travel at 65 mph, what radius should they specify? [[15]](#footnote-15)

Solution

**Identifying the variables**

* is the speed of the car in mph
* is the radius of the curve in feet

### Setting Up the Problem

### Final Answer

The radius of the curve should be 2010 feet.

# Articles and Videos

## Pollution

<https://www.wnyc.org/story/decades-deceit-extent-grummans-long-island-pollution/>

## Setting speed limits

<https://www.fhwa.dot.gov/publications/publicroads/13sepoct/02.cfm>

## Cyclist corner turn

<https://www.youtube.com/watch?v=hRM3bFXlyNk>

<https://www.youtube.com/watch?v=aPsVl42tqYo>

# Appendix

* Pollution Plume handout
* Velocity vs Height Handout
* Radical Practice Problems – Evaluating Handout
* Water Storage Handout
* Radical Practice Problems – Equations Handout

1. McKaegue, Charles P. Intermediate Algebra, eighth edition. Thomson Brooks/Cole. 2008. pg 407, 452. [↑](#footnote-ref-1)
2. <https://www.nationalgeographic.org/encyclopedia/point-source-and-nonpoint-sources-pollution/> [↑](#footnote-ref-2)
3. Jeanne Bendick, Along came Galileo (Beautiful Feat Books, Sandwich, MA, 1999 [↑](#footnote-ref-3)
4. Geoffrey Akst, Sadie Bragg. Introductory Algebra through Applications. Pearson/Addison Wesley. 2009.Pg. 590, 611. [↑](#footnote-ref-4)
5. Robert Blitzer. Introductory Algebra for College Students. Prentice – Hall, Inc. 1998. pg. 641. [↑](#footnote-ref-5)
6. https://sciencing.com/humidity-wind-speed-affect-evaporation-12017079.html [↑](#footnote-ref-6)
7. Ibid. pg. 645. [↑](#footnote-ref-7)
8. Ibid. pg. 639, 645. [↑](#footnote-ref-8)
9. Angel, Allen R. and Dennis c. Runde. Elementry and Intermediate Algera, 4th edition. Prentice Hall.2011. pg. 653. [↑](#footnote-ref-9)
10. Modified from: Alan S. Tussy, R. David Gustafson. Elementary Algebra. Thomson Brooks/Cole. 2008. Pg 720. [↑](#footnote-ref-10)
11. Robert Blitzer. Introductory Algebra for College Students. Prentice – Hall, Inc. 1998. pg. 685 [↑](#footnote-ref-11)
12. Modified from: Angel, Allen R. and Dennis c. Runde. Elementry and Intermediate Algera, 4th edition. Prentice Hall.2011.Pg. 662. [↑](#footnote-ref-12)
13. Angel, Allen R. and Dennis c. Runde. Elementry and Intermediate Algera, 4th edition. Prentice Hall.2011. pg. 639 [↑](#footnote-ref-13)
14. Robert Blitzer. Introductory Algebra for College Students. Prentice – Hall, Inc. 1998. pg. 679. [↑](#footnote-ref-14)
15. Alan S. Tussy, R. David Gustafson. Elementary Algebra. Thomson Brooks/Cole. 2008. Pg 719. [↑](#footnote-ref-15)