# Introduction

This segment intends to help students understand the explosive power of an exponential expression. It is not a segway into the study of exponential and logarithmic functions but instead an introduction into exponents and laws of exponents. Two activities are included.

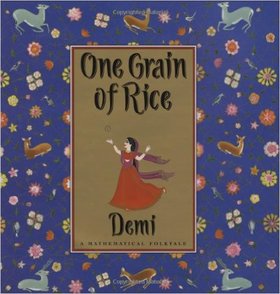
For an on-ground or remote class, assign “One Grain of Rice” as reading and follow up with the Covid-19 activity in class.

For an online class you can use either one as a standalone assignments.

The appendix includes a file with the story and a link to a video version for the reading assignment. There is another file with the story, the link and and the student activity embedded.

# Activity for Connection 1: One Grain of Rice

<https://www.youtube.com/watch?v=vfH8lQwBHfQ>

[](https://www.mathsthroughstories.org/uploads/5/7/2/5/57253055/5061100_orig.jpg)

* Number of pages: 40
* Publisher: Scholastic Press (USA)
* First published in: 1997
* Format: Picturebook

Long ago in India, there lived a raja who believed that he was wise and fair. But every year he kept nearly all the people’s rice for himself. Then a village girl named Rani devises a clever plan, using the surprising power of doubling to win more than one billion grains of rice from the raja. [[1]](#footnote-1)

## Materials list:

### For Each Student

* One Grain of Rice Handout (see Appendix)

## Discussion Questions

* Why was the raja surprised by the request and outcome?
* Why did it take him so long to see the consequences of the agreement?

The answer to the last question is the same reason many countries’ response to the COVID-19 pandemic was delayed. An understanding of the trend in exponential growth was vital to making sound decisions by public health professionals.

# Activity for Connection 2: Pandemic

At the start of the COVID-19 pandemic, it was estimated that every infected person would infect four others. The incubation period was often around 5 - 6 days but could take up to 14. [[2]](#footnote-2) Assuming an incubation period of one week (7 days), how long would it take for 3.5 million people (the population of CT) to be infected?

Note: This is the simplest model for infection rate. Many scientists, immunologists and mathematicians worked to define and refine models to make accurate predictions for infection rates as more data on the factors associated to contagion spread became available.

A picture containing chart

Description automatically generated

|  |  |  |
| --- | --- | --- |
| Week | Number of new cases each week | Total number infected |
| 1 | 1 | 1 |
| 2 | 4 | 5 |
| 3 | 16 | 21 |
| 4 | 64 | 85 |
| 5 | 256 | 341 |
| 6 | 1,024 | 1,365 |
| 7 | 4,096 | 5,461 |
| 8 | 16,384 | 21,845 |
| 9 | 65,536 | 87,381 |
| 10 | 262,144 | 349,525 |
| 11 | 1,048,576 | 1,398,101 |
| 12 | 4,194,304 | 5,592,405 |
| 13 | 16,777,216 | 22,369,621 |
| 14 | 67,108,864 | 89,478,485 |
| 15 | 268,435,456 | 357,913,941 |
| 16 | 1,073,741,824 | 1,431,655,765 |

You may have noticed that the number of new cases are powers of 4. To find the number of new cases for any week, use the equation where y is the number of new cases and x is the number of weeks since the first case. For a video discussion of COVID-19 and exponential spread see: [**https://www.youtube.com/watch?v=fgBla7RepXU**](https://www.youtube.com/watch?v=fgBla7RepXU)

Both the following discussion questions as well as the questions to extend the lesson can be used for classroom discussion, small groups for on-ground classes, breakout rooms for live remote classes, or on the classroom discussion board for online classes

## Materials list:

### For Each Student

* Pandemic Handout (see Appendix)

## Discussion Questions

* When the media and public officials talked of “flattening the curve”, what did that statement refer to?
* What was your response to COVID-19 and your attitude toward local and state-wide mandates on shutdowns and restrictions on activities? Would an understanding of exponential growth have changed your actions?
* Can you think of any other situations where applying knowledge of exponents might deepen your understanding?

### Answer:

When chemists indicate a substance has a pH of 7, they know this represents 107 while a substance with a pH of 8 represents 108. This means that the substance with the pH of 8 is 10 times more basic than the substance with the pH of 7.

Geophysicists also use a logarithmic scale. An earthquake that measures a 7 on the Richter scale clocks in at 107 for seismic energy while an earthquake measuring an 8 represents 108 for seismic energy. This means the second earthquake is 10 times more powerful than the first. [[3]](#footnote-3)

# Extending the Lesson: Jack Reacher

There is plenty of math in Lee Child’s Reacher novels to use as a discussion point in class or to return to group work. Have students find the reference to exponents and present a mathematical explanation to confirm or refute Reacher’s argument.

## Materials list:

### For Each Student

* Jack Reacher Handout (see Appendix)

## Excerpt 1

*OK, now we’ll go break into his house.” Which we did by kicking down the door. Which was easy enough. A question of force, obviously, which is [proportional to] the product of mass times velocity squared, and that squared part puts a premium on speed, not weight. Bulking up by twenty pounds at the gym is good, because it throws and extra twenty pounds in the mix but moving your foot an extra twenty percent faster is better. It does you four hundred percent of a favor. Because it gets squared. Which means multiplied by itself. Money for nothing. Like in baseball. You can swing a heavy bat slow or a light bat fast, and the slow heavy bat gets you a high fly to the warning track, and the light fast bat puts the ball in the bleachers.* [[4]](#footnote-4)

### Algebra behind the activity:

is original force, is original mass, is original velocity

add 20 pounds:

add 20% speed:

## Excerpt 2

*Three hours in a car? I said, “At night, with empty roads? Anything up to a two-hundred-mile radius. A hundred and twenty-five thousand square miles. Approximately. Pi times the radius squared. [[5]](#footnote-5)*

### Algebra behind the activity:

Assumes a speed of or approximately 67. The area of a circle is given by .

# Practice Exercises – Characteristics of Exponents

## Deforestation

The number of square miles per year of rain forest destroyed in Brazil is given by , where x is the number of years from 1990.

1. With this model, how many square miles were destroyed in 1990?
2. With this model, how many square miles will be destroyed this year? [[6]](#footnote-6)

### Answer

1. For Year 1990

4900 square miles were destroyed in year 1990

1. For year 2021

About 22,236 square miles will be destroyed in year 2021

## Drugs in the Bloodstream

The concentration of a drug in the bloodstream from the time a drug is injected until 10 hours later is given by in which *y* is the percent concentration *t* hours after injection. What is the percent concentration after 5 hours. [[7]](#footnote-7)

### Answer

The concentration is about 83% after 5 years

## Bird Population

A bird species in danger of extinction has a population that is decreasing exponentially. Five years ago the population was 1400 and today only 1000 of the birds are alive represented by an exponential model defined by in which is the number of years from five years ago. Once the population drops below 100, the situation will be irreversible. Will this happen within 20 years? 40 years? [[8]](#footnote-8) [[9]](#footnote-9)

### Answer

In 20 years (from five years ago)

Does not happen in 20 years

In 40 years (from five years ago)

Within 40 years (35 years from now) the population drops below 100

## A bit of a riddle

There is a pond with one **lily pad** in it. Every day the amount of **lily pads** in the pond doubles. If the pond is completely covered on the 30th day, on which day was it half full? [[10]](#footnote-10)

### Answer

Since the lily pads double each day the pond will be half full of lily pads by day 29.

# Practice Exercises – Laws of Exponents

## Radioactive decay

When the tsunami caused by the 2011 Tohoku earthquake hit Japan, radioactive caesium-137 was released. Caesium-137 has a half-life of 30 years; that is ½ of what was released that day will remain after 30 years. In another 30 years the amount will be reduced by ½ again. Calculations suggest it will take 19,293,719,920 seconds (about ) to reduce the amount of Caesium-137 to safe levels. To approximate the number of years to reach safe levels, multiply by using scientific notation. [[11]](#footnote-11)

### Answer

It takes 600 years for the amount of Caesium-137 to reach the safe level.

## Space travel

Just how long it will take to get to Mars depends on the location of both the earth and mars in their current orbits and the spacecraft’s cruising speed. With a cruise speed of 24,600 mph and a distance of 292,000,000 miles, use scientific notation to calculate the number of days it will it take to travel to Mars. [[12]](#footnote-12) [[13]](#footnote-13)

### Answer

Speed:

Distance:

Time:

To convert to days

It takes about 495 days to reach the Mars.

## Growing Bacteria

The equation approximate the number of bacteria in a certain culture after hours. Keep your answers in exponential form and use the laws of exponents, not a calculator.

1. The initial number of bacteria is determined when . What is the initial number of bacteria?
2. How many bacteria are there after 1 hour, 2hours, and 8 hours. [[14]](#footnote-14)

### Answer

The initial number of bacteria is



After 1 hour

There are bacteria after 1 hour

After 2 hour

There are bacteria after 1 hour

After 8 hours

There are bacteria after 8 hour

## Earthquakes

The expression  represents how many times stronger an earthquake measuring 7.8 on the Richter scale (2010 Indonesia) is than one measuring 7.0 Rither (2010 Haiti). The difference in energy released is represented by . Simplify both expressions using the properties of exponents. [[15]](#footnote-15)

### Answer

It is about 6.3 times stornger

About 15.8 time more energy released

# Article and video links

Exponent rules

<https://www.youtube.com/watch?v=V6yixyiJcos>

Numberphile

<https://www.youtube.com/watch?v=k6nLfCbAzgo>

<https://www.youtube.com/watch?v=Kas0tIxDvrg>

Deforestation

<https://www.nytimes.com/2020/06/06/world/americas/amazon-deforestation-brazil.html?searchResultPosition=1>

# Appendix

* One Grain of Rice Story Student Handout
* One Grain of Rice Story and Activity Sheet Student Handout
* Pandemic Student Handout
* Jack Reacher Student Handout
* Exponenetial Expression Practice Problems

1. <https://www.mathsthroughstories.org/one-grain-of-rice.html> [↑](#footnote-ref-1)
2. <https://www.medicalnewstoday.com/articles/covid-19> [↑](#footnote-ref-2)
3. <https://sciencing.com/> [↑](#footnote-ref-3)
4. Personal, Lee Child, 2014 pg 52 [↑](#footnote-ref-4)
5. The Enemy, Lee Child pg 290 [↑](#footnote-ref-5)
6. Modified from: Harshbarger, Ronald J. and Lisa S. Yocco. College Algebra in Context, 4th Edition. Pearson. 2013. pg 353. [↑](#footnote-ref-6)
7. Modified from: Harshbarger, Ronald J. and Lisa S. Yocco. College Algebra in Context, 4th Edition. Pearson. 2013. pg 353. [↑](#footnote-ref-7)
8. Modified from: Blitzer, Robert, Introductory and Intermediate Algera, 5th edition. Pearson Education.2017. pg. 921 [↑](#footnote-ref-8)
9. <https://portal.ct.gov/DEEP/Endangered-Species/Endangered-Species-Listings/Endangered-Threatened--Special-Concern-Birds> [↑](#footnote-ref-9)
10. <https://www.youtube.com/watch?app=desktop&v=fgBla7RepXU> (start at 2:30 mark in video) [↑](#footnote-ref-10)
11. McKell, Melinda Kathryn (2014) “Fukushima: A Model Study,” Undergraduate Journal of Mathematical Modeling: One + Two: Vol. 6: Iss. 1, Article 6. Available at: <https://scholarcommons.usf.edu/ujmm/vol6/iss1/6> [↑](#footnote-ref-11)
12. <https://www.jpl.nasa.gov/edu/learn/video/mars-in-a-minute-how-do-you-get-to-mars/> [↑](#footnote-ref-12)
13. <https://mars.nasa.gov/mars2020/timeline/cruise/> [↑](#footnote-ref-13)
14. Modified from: Angel, Allen R. and Dennis c. Runde. Elementry and Intermediate Algera, 4th edition. Prentice Hall.2011. pg. 639 [↑](#footnote-ref-14)
15. <https://www.youtube.com/watch?v=YXMKSOsv3QA> [↑](#footnote-ref-15)