# Introduction

I prefer to use an activity in context at the start of each unit so that the problem can be referred to throughout the unit. Since this activity is not based on prior knowledge of rational expressions and can be done with arithmetic and skills of observation, it can be used at the start of the unit on Rational Expressions.

# **Activity for Connection** – Climate Change Intro to Rational expressions

## The Question

A picture containing nature

Description automatically generatedSuppose the annual cost (in hundreds of dollars) of removing *p*% of the particulate pollution from the smokestack of a power plant is given by . Is it possible to remove all the pollution?

## The process:

The activity will take the student through the process of calculating the costs for specific amounts of pollution and then through analysis of what those data points actually mean. The students will notice that the cost gets prohibitively expensive as the percent removed gets closer to 100% and that no value is possible with this equation at 100%. This illustrates restricted values.

|  |  |
| --- | --- |
| *p* |  |
| 20 |  |
| 40 |  |
| 60 |  |
| 80 |  |
| 85 |  |
| 90 |  |
| 95 |  |

1. Complete the table.

Interpret the meaning of each row (ordered pair).

|  |
| --- |
|  |
|  |
|  |
|  |
|  |
|  |
|  |
|  |

1. The rate of change between two points is defined by the slope of the line that connects the two points. Find the slope between each successive set of points.

Looking at the completed tables, what conclusion can you draw about the cost of removing particulate pollution?

1. Suppose the annual fine for removing less than 80% of the particulate pollution from the smokestack is $700,000. If you were the company’s Chief Financial Officer, would you advise the company to remove the pollution or pay the fine? Why?
2. If the company has already paid to remove 60% of the particulate pollution, would you advise the company to remove another 20% or pay the fine?
3. If you were a lawmaker, to what amount would you raise the fine to encourage power plants to comply with the pollution regulations?
4. Compare the cost of removing 80% and 90% of the pollution from the smokestack. Do you think the benefit of removing 90% of particulate pollution rather than 80% is worth the cost?

## Materials list:

### For Each Student

* Climate Change handout (see Appendix)

## Discussion Questions

Is it possible to remove 100% of the particulate pollution?

NO. The cost is undefined, in other words, infinite, with this model. A table can be displayed using a graphing calculator with possible x values of 99 – 100 with a delta of 0.1

Can you think of other situations where the possible values are limited?

## The algebra behind the activity

### Identifying the variables

In the equation, .

* *C* is the cost in hundreds of dollars to remove particulate pollution
* *p* is the percent of particulate removed

### Setting up the Problem

|  |  |
| --- | --- |
| *p* | (hundreds of dollars) |
| 20 | 605 |
| 40 | 1613 |
| 60 | 3630 |
| 80 | 9680 |
| 85 | 13,713 |
| 90 | 21,780 |
| 95 | 45,980 |

1. Complete the table to get a sense of how cost increases in relation to the percent of pollution removed.

|  |
| --- |
|  |
|  |
| 50 |
| 101 |
| 302 |
| 807 |
| 1613 |
| 4840 |

1. The relationship between and is not linear. It’s not exponential either but the rate of change in cost is increasing. The rate of change increases with every interval so that the cost increase to remove 1% more increases as the percent already removed increases.

1. Since it cost $968,000 per year to remove 80% of the particulates and only $700,000 per year for a fine, a CFO might to choose to advise the company to pay the fine. However, the company might decide that the additional $268,000 was not a lot of money for the goodwill of their clients and community that it might buy.
2. The difference in cost between the cost of removing 80% and 60% is $605,000. This is less then the fine so the CFO will advise the removal of the additional 20% particulate pollution.
3. The cost to move from 85% to 90% removal is $806,700 but to move from 90% to 95% is over 2 million dollars ($2,420,000). I would set the fine to $1.5 million dollars to make it financially beneficial to remove at least 85% of the pollution.
4. The additional cost to move from 80% to 90% removal is $1,210,000 which more than the total current cost of removing the first 80%. Annual expenses to remove particulate pollution would more than double.

# Extending the Lesson - Restricted values

## Introduction:

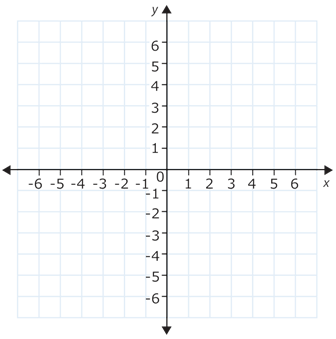
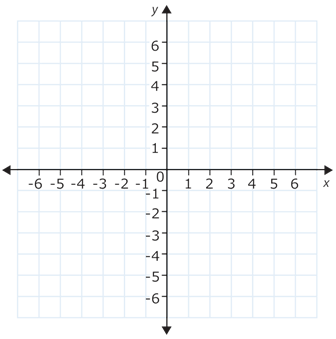
These exercises do not reflect a “real world” application but do help to see the impact of an expression in the denominator. If this exercise is used after the students learned how to simplify rational expressions, the first equation can be presented as .

# Part 1

## The Problem:

Compare the equations: [[1]](#footnote-1)

1. Do you expect the graphs to look the same? Why?
2. Look at the two graphs. For what values of *x* does and represent the same number?



1. Can you find a reasonable explanation for what you observe?

## Materials list:

### For Each Student

* Restricted Values handout (see Appendix)

## The algebra behind the activity

1. Yes. If the common factors of the numerator and denominator are reduced the two equations are the same.
2. As expected, the graphs look the same except at the point . There is no output value for the first equation when but there is a value for the second.
3. The denominator for the first equation when is zero and no value or expression can be divided by zero.

## Conclusion:

The two equations may simplify to the same expression but they started out modeling two different situations.

# Part 2

### The Problem: Proof for 2 = 0

We know does not equal , but here’s the proof. Find the fault in this argument.[[2]](#footnote-2)

(1) Suppose that *a* and *b* are any equal real numbers.

(2) Square both sides of the equation.

(3) Subtract

(4) Multiply both sides by 2

(5) On the right side,

(6) Factor

(7) Divide both sides by

(8) Divide both sides by

## .The algebra behind the activity

Step 7: since , and no value or expression can be divided by zero

# Practice Exercises – Rational Expressions

As with the Activity For Connection, the solutions to these exercises are based on evaluating expressions and making observations. They are presented again as problems to be resolved by solving equations with rational expressions.

## Health

A pharmaceutical company claims that the concentration of a drug in a patient’s bloodstream will be at least 10% for 8 hours. Clinical tests show that the concentration, *C*, of the drug (as percent) *t* hours after injection is .

1. During what time period is the concentration at least 10%?
2. Is the company’s claim supported by the data? [[3]](#footnote-3)

### Identifying the variables

* is the concentration of the drug
* is time in hours

### Setting Up the Problem

|  |  |
| --- | --- |
|  |  |
| 1 | 5.9 |
| 2 | 10 |
| 4 | 12.5 |
| 8 | 10 |
| 10 | 8.6 |

1. At this point, there is no expectation that students know how to solve an equation with rational equations, so a table of values can be used.

### Final Answer

The company’s claim is not supported by this data model. A concentration of 10% is reached in the second hour and goes below that after the 8th hour, which is six hours not eight.

## Public Health

The rational expression describes the cost, in millions of dollars, to inoculate percent of the population against a particular strain of flu.

1. Evaluate the expression for , and . Describe the meaning of each evaluation in terms of percentage inoculated and cost.
2. What happens to the cost as approaches 100%? How can this observation be interpreted? [[4]](#footnote-4)

### Identifying the variables

* is the percent of the population inoculated against the flu
* The result is the cost in millions of dollars

### Setting Up the Problem

1. When the cost is or $86,870,000 .

When the cost is or $520,000,000

When the cost is or $1,170,000,000

1. The cost is undefined at .

### Final Answer

It is not possible to inoculate 100% of the population.

## Species Extinction

To restore the population of tule elk at Point Reyes, California, 50 elk are introduced into a wildlife preserve. The tule elk population () after years is described by the model

|  |  |
| --- | --- |
| *t* |  |
| 1 |  |
| 3 |  |
| 5 |  |
| 10 |  |
| 25 |  |

.

1. Complete the table to estimate when the population will reach 125?
2. Is there a limit on the size of the population? Explain. [[5]](#footnote-5)

### Identifying the variables

* is the population after years
* is time in years

### Setting Up the Problem

|  |  |
| --- | --- |
| *t* |  |
| 1 | 76 |
| 3 | 125 |
| 5 | 166 |
| 10 | 250 |
| 25 | 400 |

1. : The denominator will never be zero as time is never negative (we can’t go back in time).

### Final Answer

1. 125
2. There is no limit. This brings up the question of the model’s accuracy in terms of limiting factors such as predators or availability of food, or perhaps the next question of what to do when the population reaches a desired number.

# Practice Exercises - applications using Rational Expressions

These exercises use all the skills necessary to master rational expressions: understanding their characteristics, performing operations, and solving equations. The exercises indicated with a ✔ were presented in exercises introducing rational expressions but solved by evaluating the expressions at specified values. Here they will require solving an equation or inequality.

## Modelling using Rational Equations: Health

✔

A pharmaceutical company claims that the concentration of a drug in a patient’s bloodstream will be at least 10% for 8 hours. Clinical tests show that the concentration, *C*, of the drug (as percent) *t* hours after injection is .

1. During what time period is the concentration at least 10%?
2. Is the company’s claim supported by the data? [[6]](#footnote-6)

### Identifying the variables

Solution

and

and

* is concentration of the drug
* is time in hours

### Setting Up the Problem



### Final Answer

The company’s claim is not supported by this data model. A concentration of at least 10% is between the second hour and the 8th hour, which is six hours not eight.

✔

## Modelling using Rational Equations: Public Health

The rational equation describes the cost, in millions of dollars, to inoculate percent of the population against a particular strain of flu.

1. Find and interpret the restricted values for
2. What percentage of the population could be inoculated for 1 billion dollars [[7]](#footnote-7)

### Identifying the variables

* is the percent of the population inoculated against the flu
* is the cost in millions of dollars

Solution

### Setting Up the Problem

1. One billion is 1000 million so solve

### Final Answer

It is not possible to inoculate 100% of the population.

## Modelling using Rational Equations: Species Extinction

✔

To restore the population of tule elk at Point Reyes, California, 50 elk are introduced into a wildlife preserve. The tule elk population () after years is described by the model

.

When the population will reach 125?

### Identifying the variables

Solution

* is the population after years
* is time in years

### Setting Up the Problem

.

### Final Answer

The population will be 125 after 3 years.

This problem brings up the interesting question of the model’s accuracy in terms of limiting factors such as predators or availability of food, or perhaps the next question of what to do when the population reaches a desired number.

## Modelling using Rational Equations: Water Quality

In Palo Alto, California, a government agency ordered computer-related companies to contribute to a pool of money to clean up underground water supplies. (The companies had stored toxic chemicals in leaking underground containers.) The formula models the cost, , in millions of dollars, for removing percent of the contaminants.

1. What percentage of the contaminants can be removed for $2 million?
2. What percentage of the contaminants can be removed for $8 million?

Solution

1. Is it possible to remove all of the contaminants? Explain the reasoning behind your answer.

### Identifying the variables

* is the cost in millions of dollars
* percent of contaminant removed

### Setting Up the Problem

1. is a restricted value

### Final Answer

1. With this model there is no amount of money that would remove 100% of the contaminant.

## Proportions: Tracking Populations

The Pribilof Islands in Alaska supports about half of the global population of Northern fur seals. St. Paul Island, the largest of the Pribilof Islands, has 12 fur seal rookeries (breeding places). In 1961, to estimate the fur seal pup population in the Gorbotch rookery, 4963 fur seal pups were tagged in early August. In late August, a sample of 900 pups was observed and 218 of these were found to have been previously tagged. Estimate the total number of fur seal pups in this rookery.

### Identifying the variables

* is the total number of pups in the Gorbotch rookery

### Setting Up the Problem

Solution

### Final Answer

pups in the Gorbotch rookery (rounded to nearest whole number)

This problem has some extraneous data (12 rookeries on St. Paul Island)

## Proportions: Drug Dose

*If a hospitalist places an order for your instructor to receive 14 mg of the same medication, should the nurse comply with the order?*

A medication is to be given to a patient at a rate of 2.8 mg for every 40 pounds of body weight. How much medication should be given to a patient who weighs 190 pounds?

### Identifying the variables

Solution

* is the number of milligrams given to a patient of

a specific weight

### Setting Up the Problem

### Final Answer

will be given to a patient weighing 190 lbs.

## Proportions: Similar Triangles

A firefighter needs to estimate the height of a burning building. She estimates the length of her shadow to be 8 feet and the length of the building’s shadow to be 60 feet. Find the approximate height of the building if the firefighter is 5’ tall.

### Identifying the variables

Solution

* is the height of the building

### Setting Up the Problem

;

### Final Answer

The building is

## Distance: Safe Driving Speed

By increasing your speed by 10 mph, you can drive the 200-mile trip to your hometown in 40 minutes less time than the trip usually takes. How fast do you usually drive?

### Identifying the variables

**Is the 40 minutes’ worth the risk?**

Charles Farmer, Insurance Institute for Highway Safety vice president for research and statistical services, found that a 5 mph increase in the maximum speed limit was associated with an 8 percent increase in the fatality rate on interstates and freeways. [www.iihs.org](http://www.iihs.org)

* is normal driving speed

### Setting Up the Problem

so

### Final Answer

Normal driving speed is

Solution

*conversion:*

*speed:*

and

and

## WorK: Natural disasters

As the spring flood waters began to recede in Texas, a young family was faced with pumping the water from their basement. One pump they were using could dispose of 9000 gal in 3 hours. A second pump could dispose of the same number of gallons in 4.5 hours. How many hours would it take to dispose of 9000 gal if both pumps were working?

9000 GALLONS IS APPROXIMATELY 1200 CUBIC FEET. iF THEIR BASEMENT IS 40’ X 20’ AND THE WATER ROSE TO 3.5 FEET, HOW LONG WILL THE TWO PUMPS HAVE TO WORK?

### Identifying the variables

Solution

* is the total time in hours to pump the basement using both pumps

### Setting Up the Problem

### Final Answer

With the two pumps working together it would take or

# Articles and Videos

<https://www.youtube.com/watch?v=DK1uKpJ49Jo>

<https://www.youtube.com/watch?v=-UM2GXbCIAw>

<https://www.youtube.com/watch?v=fZsw3j5x4KI>

<https://www.youtube.com/watch?v=XR9QDz70DaI>

<https://www.youtube.com/watch?v=7y9Cn60h7Ug>

<https://www.youtube.com/watch?v=a0heBE3IuOw>

<https://www.c2es.org/content/drought-and-climate-change/>

[Group of Friends Track Block Island’s Crumbling Shoreline — ecoRI News](https://www.ecori.org/shifting-sands/2021/6/17/group-of-friends-track-block-islands-eroding-shoreline)

Cool Careers in Science – PBS Learning Media

<https://cptv.pbslearningmedia.org/resource/biot09.biotech.car.drennan/career-profile-chemist-and-biologist-catherine-drennan/> (global warming – creating organism to consume carbon dioxide)

# Appendix – Student Handouts

* Climate Change – Intro to Rational Expression
* Restricted Values
* Rational Expressions Practice
* Rational Applications Practice

1. Robert Blitzer. Introductory Algebra for College Students. Prentice – Hall, Inc. 1998. pg 471. [↑](#footnote-ref-1)
2. Robert Blitzer. Introductory Algebra for College Students. Prentice – Hall, Inc. 1998. pg 530 [↑](#footnote-ref-2)
3. Harshbarger, Ronald J. and Lisa S. Yocco. College Algebra in Context, 4th Edition. Pearson. 2013. Pg 491. [↑](#footnote-ref-3)
4. Robert Blitzer. Introductory and Intermediate Algebra for College Students. Pearson Education,Inc. 2017. (pg 498) [↑](#footnote-ref-4)
5. Robert Blitzer. Introductory Algebra for College Students. Prentice – Hall, Inc. 1998. [↑](#footnote-ref-5)
6. Harshbarger, Ronald J. and Lisa S. Yocco. College Algebra in Context, 4th Edition. Pearson. 2013. Pg 491. [↑](#footnote-ref-6)
7. Robert Blitzer. Introductory and Intermediate Algebra for College Students. Pearson Education,Inc. 2017. (pg 498) [↑](#footnote-ref-7)