The University of Alabama at Birmingham

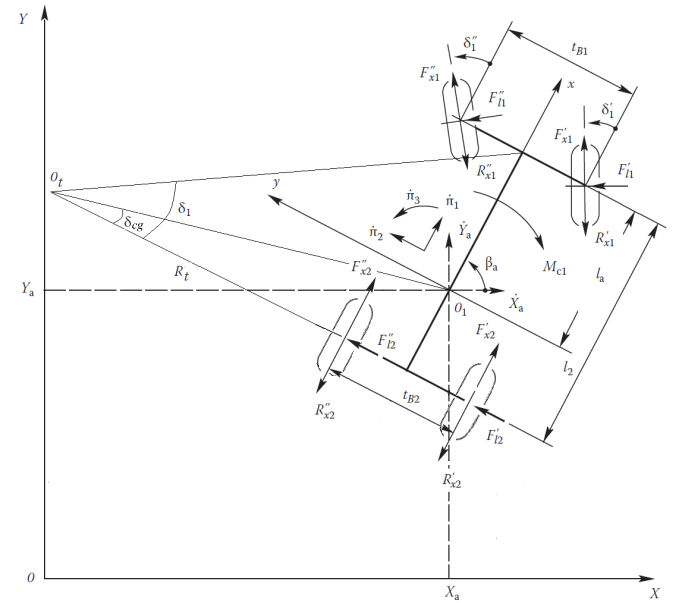
School of Engineering

Department of Mechanical Engineering

in Collaboration with

Center for Advanced Automotive Technology

Virtual Test of a 4x4 Hybrid Electric Vehicle



***Prepared By:***

**Vehicle and Robotics Engineering Laboratory**

**[](http://www.uab.edu/engineering/home)Mechanical Engineering Department**

**School of Engineering**

**University of Alabama at Birmingham, USA**

**2017**

**Vladimir V. Vantsevich**

**Professor and VREL Director**

**Jesse R. Paldan**

**Research Assistant**

# Table of contents

* Objective ……………………………………………………………..…………………...3
* Vehicle Model Overview..……………………………...………………………………....3
* 4x4 Vehicle Math and Computer Model…………………………………...…...…..……..8
* References ….……………………………………..…………………….……..………...22
* Assignment ……………………………………………………………...….……………23

## Objective

1. Understand the modeling equations of a 4x4 vehicle
2. Perform a virtual test of a 4x4 vehicle in a fishhook maneuver

### Vehicle Model Overview

##### 1.1 4x4 Vehicle

In this project, a virtual test of a hybrid electric 4x4 vehicle is performed in which the vehicle is subjected to a fishhook test. The simulation includes a mathematical model of a vehicle in planar motion, allowing the vehicle’s performance of maneuvers to be evaluated at all four wheels. The vehicle’s driveline incorporates open differentials which split torques between the axles and to each wheel. The hybrid electric powertrain includes a model of the electric motor, battery, engine, and generator.

When a vehicle is turning, the paths traveled by each wheel are not the same. This requires each of the wheels to move at different velocities. As seen in Fig. 1, the steer angle differs between the left and right wheels, giving their paths different curvatures. Velocity on the left rear wheel is different from velocity of the right rear wheel and also different from velocity , the actual linear velocity of the axle center. Other factors that cause differences in the velocities are inequalities in the tire radii, inflation pressures, or wheel loads [1].

Inequality in the velocities of the driven wheels can be accommodated by using open differentials, a power dividing unit with a single input link and two output links. Differentials can be interaxle, dividing power between axles, or interwheel, dividing power between the left and right wheels of an axle. Figure 2 shows an interwheel differential. The input axle torque is divided into two output torques and ; the relationship between the input and output torques is therefore . An open differential can be either symmetrical or asymmetrical. In a symmetrical differential, while for an asymmetrical differential . The relationship between the torques can be expressed by introducing the differential’s internal gear ratio, , in which . If the differential is symmetrical, .

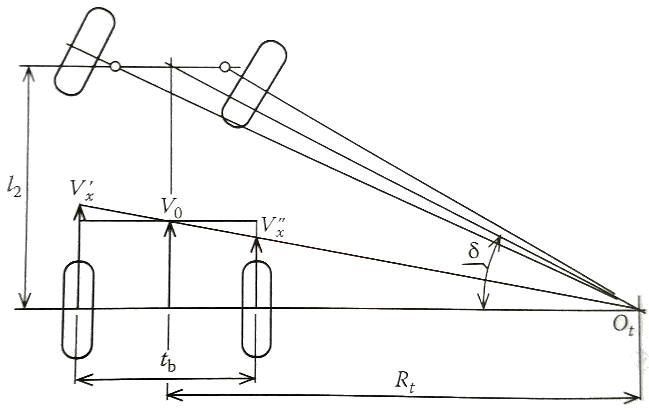


Figure 1: Vehicle in turn with wheels making different paths of travel [1]

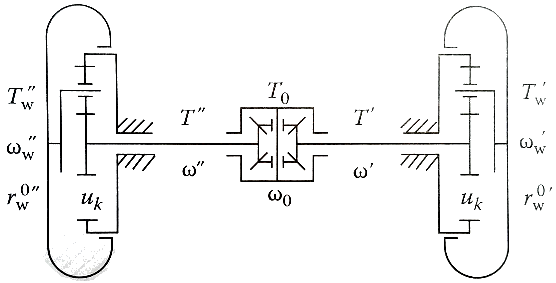


Figure 2: Interwheel open differential [1]

##### 1.2 Additional Simulink Functionality

To understand the 4x4 model, several additional functions of Simulink used in the model need to be explained first. Vector signals are signals which contain a vector of multiple elements rather than a single number. Fig. 3 shows how vectors are used in calculations. In (A), the constant is multiplied by a gain. This is a standard scalar signal of one element. In (B), the same constant is multiplied by a gain of two elements. The constant is multiplied by each gain, creating a 2-element vector whose values are 10 and 15. In (C), a two-element vector is passed through a gain of 2 elements. The gains are applied element-by-element, so in the result, 5 is multiplied by 2 and 1 is multiplied by 3. A mux block (D) collects multiple signals into a vector. A demux block (E) breaks a vector into individual elements.

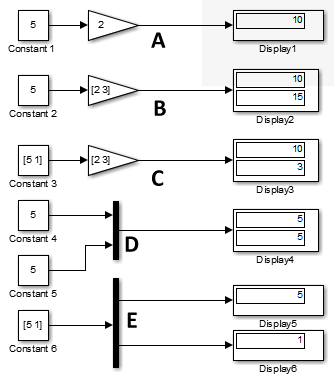


Figure 3: Vector Signals

Another type of signal grouping is a bus signal. Bus signals are used to collect multiple related signals in a group (Fig. 4). The bus creator (right) takes several named signals, which can be vectors or scalars, as inputs. The bus selector (right) extracts one or more signals from the group. Signals may be chosen by double-clicking the bus selector. A bus signal is depicted as a thicker line than standard signals.

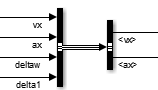


Figure 4: Bus signals

A subsystem allows a complex model to be organized by moving calculations into separate blocks called subsystems. Therefore, the model can be created in layers with parts of the model divided into smaller systems of related calculations. Fig. 5A is the outside of the subsystem, which contains inputs and outputs. Fig. 5B is the interior of the subsystem which may be accessed by double-clicking it, showing its inports and outports. In this example, input In1 enters the subsystems, is multiplied by a gain of 2, and exits the subsystem as output Out1.

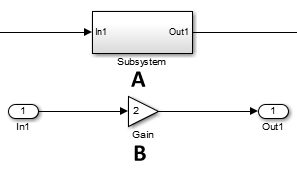


Figure 5: Subsystem

A lookup table (Fig. 6) calculates its output value by looking up (or interpolating) a table of values from its input.

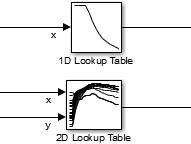


Figure 6: 1 dimensional and 2 dimensional lookup tables

In effect, it allows a piecewise curve to be defined by entering a sequence of output values which correspond to a second sequence of input values. When the input the block receives is between the defined input values, it can be set to interpolate the result or use the nearest value. The table can be 1-dimensional, in which the output value is looked up from one input, or multi-dimensional, in which the result is based on multiple inputs. In the model, lookup tables are used for performance curves for various systems. For example, a motor efficiency table looks up a motor’s efficiency value from curves for motor efficiency as a function of speed and torque.

##### 1.3 Using array variables in plots

Data saved to the workspace is saved in a 2-dimensional array format. MATLAB commands for working with arrays include:

Table 1: Array commands

|  |  |
| --- | --- |
| slip(1,3) | Displays entry (1,3) of the array. In the saved data, the first dimension is the time entry, the second is the location. Therefore, slip(1,3) displays the value of slippage for the 3rd tire (right rear) at the initial time (1). |
| slip(1,1:2) | Displays slippage values at the initial time for location 1 to 2 (the front tires) |
| slip(:,1:2) | Displays all time data for the front tires |
| slip(:,:) or slip | Displays the entire contents of the variable “slip” |

Arrays may be used in plots as follows. Create a plot of all elements of tire slip (‘slip’) vs. time (‘t):

plot(t,slip)

Create a plot of multiple variables vs. time (‘t’):

plot(t,[slip(:,1) slip(:,1) omega(:,2) omega(:,2)])

Add a plot legend:

legend(‘Tire 1’,’Tire 2’,’Tire 3’,”Tire 4’)

Use special characters in a legend or title:

Subscript: slip\_1 =

Superscript: slip^1 =

Group of characters: slip\_{w1} =

Greek character: s\_{\delta} =

Create multiple plots (slip vs. time and Fx vs. time) at once:

figure(1)

plot(t,slip)

figure(2)

plot(t,Fx)

Multiple plots may be created on a single figure using the subplot command. The command

subplot(m,n,p)

will place the next plot on location p of an m by n grid of subplots. For example, if m=3 and n=2, then the figure will include 6 possible plots (3 rows and 2 columns); they can each be made by setting p = 1 to 6.

### 4x4 Vehicle Math and Computer Model

##### 2.1 Model Block Diagram

The input to the model is the driver’s steering input and parameters of the vehicle and terrain. The output is the vehicle’s motion as it follows the maneuver as well as calculations of the loads, forces, and tire slip values. The block diagram of the Simulink model is shown in Fig. 7. The model is divided into subsystems which calculate different properties of the 4x4 model.

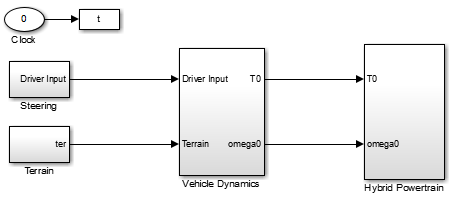


Figure 7: Model Overall Block Diagram

Each block in Fig. 7 is a subsystem containing one aspect of the model. The calculations for each subsystem are explained in this section.

##### 2.2 Driver and Terrain Input

The Driver Input block contains the steering and vehicle speed inputs. The front wheels are steered by the driver into the fishhook maneuver. This defined as angle . This angle is output by the Signal Builder block. This block defines the curves of vs. time which represent the fishhook turn’s steering maneuver made by the driver. The left and right wheels have their own angles and which differ from (see Fig. 8):

(1)

(2)

where is the distance between left and right wheels and is the distance between the front and rear axles. Subscript represents the axle number. Since the 4x4 vehicle has two axles, is evaluated for 1 to 2 in all equations.



Figure 8: Left and Right steer angles

The terrain input block includes parameters representing the ground conditions. These are the ground height , the road incline angle , peak coefficient of friction rolling resistance coefficient , and terrain parameter . These parameters have important effects on the vehicle which will be explained where they are used in later blocks.

##### 2.3 Vehicle Dynamics Subsystem

Figure 9 is the block diagram of the vehicle dynamics. Each block contains calculations which will be explained in this section.

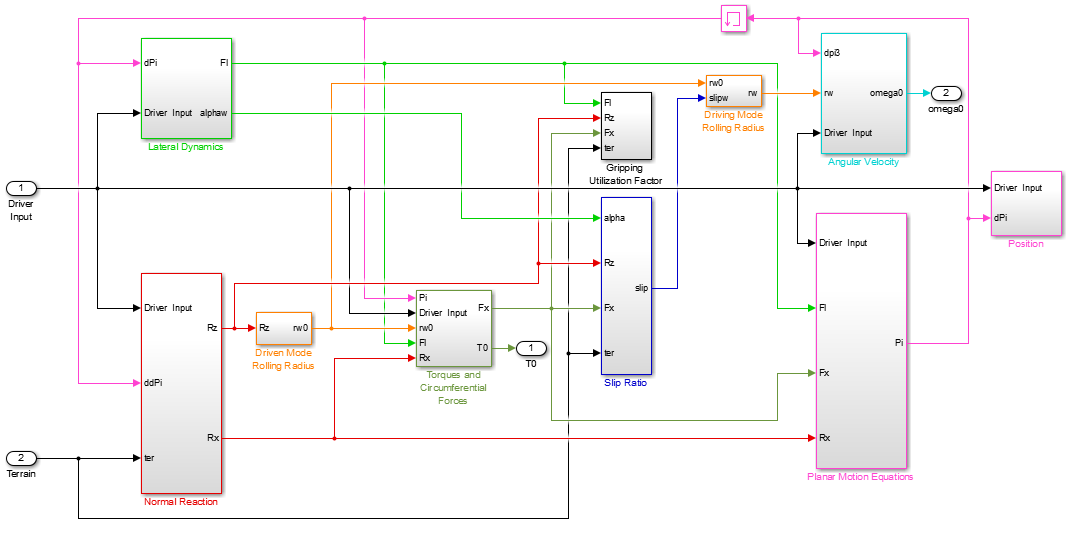


Figure 9: Vehicle Dynamics Subsystem

###### Mathematical Model of the Vehicle’s Dynamic Normal Reactions

For a 4x4 vehicle, separate normal loads and dynamic reactions can occur at each wheel. During acceleration, dynamic shifting in the normal loads take place under the influence of forces in the longitudinal direction, lateral acceleration, and vibration of the vehicle’s sprung and unsprung masses in each corner. The resultant normal loads are therefore calculated in three steps. First is the effect of longitudinal forces. For a vehicle with two axles and equal tire and suspension stiffness at the front and rear axles, the normal loads at the front and rear axles and are

(3)

(4)

where stands for so-called static reactions that are computed on a surface of motion without lateral and vertical effects. Moment is given by

(5)

is the vehicle weight, the horizontal distance from the front axle to the center of gravity, the center of gravity height, the longitudinal slope angle, the inertia force, the air drag force, and the height of the drag force.

In curvilinear motion, the loads at the left and right wheels will be different during lateral acceleration due to the lateral centrifugal forces . Equations for the left/right shift of the normal loads are determined by summing moments about the ground contact point of the left and right tire:

(6)

(7)

where ‘ and ‘’ are for the left and right wheels, is the height of the wheel gravity center. Forces act at the height of the gravity center for the front and rear wheel, loading the outer wheels and reducing the load of the inner wheels during a curvilinear maneuver. The forces are computed proportionally to the front and rear masses and the lateral acceleration of the vehicle with

(8)

The dynamic normal reactions at each wheel are calculated using a 1/4 model of vertical vibrations of the sprung and unsprung masses and applying it to each wheel. Figure 10 shows the vibration model of one corner of the vehicle.



Figure 10: Corner vibration model

is the height of the road surface, the vertical displacement of the unsprung mass , and the vertical displacement of the sprung mass . and are the stiffness and damping of the suspension. and are the stiffness and damping of the tire. Two equations describe the motion of the sprung and unsprung masses:

(9)

(10)

The displacements of the sprung and unsprung masses are used to calculate new values of the normal loads with dynamic using [1].

(11)

A higher load generates proportionally higher rolling resistance. Dynamic reactions on each tire are used to calculate rolling resistance forces :

(12)

is a coefficient of rolling resistance between the tire and ground which defines the proportionality between and .

###### Rolling Radius in the Driven Mode

Another use of the normal reaction is calculating the rolling radius in the driven mode, . This is computed from the normal loads and inflation pressure using

(13)

where is the unloaded tire radius and are empirical factors [1].

###### Planar Motion Equations

The Planar Motion block models the motion that takes place when the vehicle is taking a turn. Fig.11 shows the XY-global (fixed to the ground) and xy-moving (attached to the vehicle) coordinate systems and forces acting upon the vehicle moving in a plane.

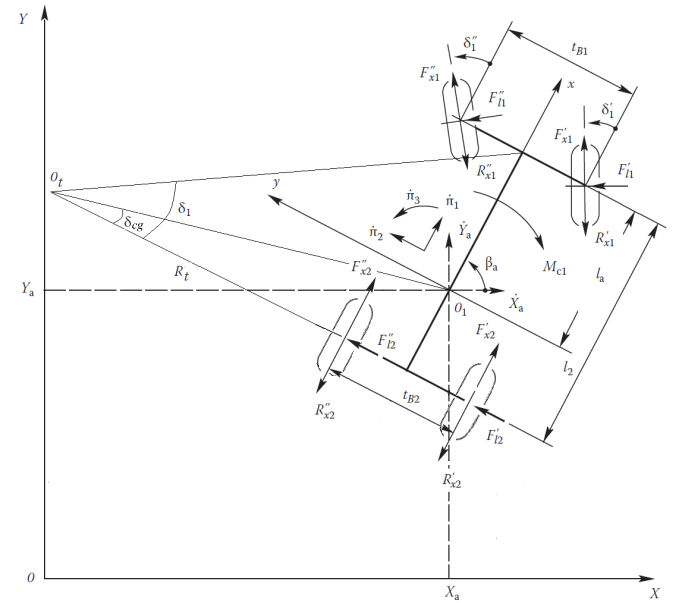


Figure 11: Vehicle making a turn [1]

The generalized coordinates and give the position of the center of mass within fixed coordinates XOY. Angle is the rotation of the vehicle’s longitudinal axis about the vertical axis. The motion is described by three quasi-velocities , , given by the following equations which relate to , and, [1].

(14)

(15)

(16)

Here, is equal to the vehicle’s forward (longitudinal) velocity, is the lateral velocity, and (or ) is the rotation rate of the center (yaw rate).

From considering the vehicle’s acceleration energy of the translation of the vehicle’s center of mass and the rotational motion about the center [1, 2], three equations are used for the vehicle in planar motion with ’ representing a right wheel and ‘’ a left wheel.

(17)

(18)

(19)

The velocity along the vehicle’s axis x is considered constant, therefore . is the distance between axle 1 and axle (). The equations include forces , , and . are circumferential forces developed by the wheel torque. Lateral forces are calculated in the Lateral Dynamics.

###### Lateral Dynamics

The forces are lateral forces which are a function of the tire’s side slip angles [1]:

(20)

is the tires’ cornering stiffness and is an individual tire’s side slip angle. The cornering stiffness coefficient is a property of the tire but also affected by the type of terrain. It determines the size of the lateral force that develops at a wheel with a side slip angle . These side slip angles are calculated using [1]

(21)

where and in the slip angle equation are projections of the velocities of the wheels on the vehicle's longitudinal and lateral axes (derived from Fig. 7).

(22)

(23)

The equations (21), (22) and (23) can be combined to the following

(24)

###### Torques and Circumferential Forces

This vehicle’s driveline uses an Open Interaxle Differential in the Transfer Case. This driveline is characterized by a torque split to both outputs of the interaxle differential according to its internal gear ratio (see Section 1). If the torque input to the transfer case from the vehicle’s motor is , the axle torques and are given by

; (25)

where is the differential’s internal gear ratio (equal to 1 for a symmetrical differential) [1]; and are the fixed gear ratios from the output shafts of the differential to the front and rear axles. With Open Symmetrical differentials used in both axles for the interwheel torque split, the left and right wheel torques are then half of the axle torque:

(26)

The total required torque at the transfer case, , is given by

(27).

The equation for is derived by using (25) – (26) to substitute for in equation (28), which is all forces projected on the x-axis. Equation (28) comes out from (17) with making equal to zero (the velocity of the vehicle in its longitudinal direction is assumed constant and thus acceleration becomes zero):

(28)

The circumferential wheel force is the torque divided by the rolling radius in the driven mode, .

(29)

###### Slip Ratio

The longitudinal tire slip can be calculated from knowing forces and . For a vehicle moving in a straight line, an exponential relationship between tire slip and circumferential force is [1]

(30)

The peak friction coefficient determines the maximum circumferential force that can be developed at the contact between the wheel and surface of motion. defines the shape of the curve at which the slip ratio relates to increasing values of . For a vehicle in turn, the side slip angles also have a contribution to the longitudinal tire slip as changes the tangential elasticity of the tire [1].. The slip function with side slip effects becomes [1]

(31)

Thus, the tire slips are found by solving Equation (31) for . In the slip ratio subsystem, the values of are calculated by setting Equation (31) equal to zero (subtracting from both sides) and using an Algebraic Constraint block. The Algebraic Constraint solves for by outputting the value which would make its input equal to zero. Therefore, the output of the Algebraic Constraint is used as and its input is the equation

(32)

###### Rolling Radius in the Driving Mode

The tire slips are needed to calculate the rolling radius in the driving mode, . This radius is smaller than the driven mode radius and can be determined from and .

(33)

###### Angular Velocities

The wheel angular velocities are computed next. Each wheel’s angular velocity is calculated with

(34)

The angular velocity comes from dividing linear velocity by the rolling radius , once the effects of turning are accounted for by introducing steer angles and yaw rate . For the Open Symmetrical Interwheel Differentials, the angular velocities of the axles, , are given by [1]

(35)

The angular velocity of the transfer case input, , is calculated with gear ratios and

(36)

###### Vehicle X and Y Position

Two values of the vehicle’s X and Y position and velocity are calculated. The theoretical position of the vehicle and yaw angle are calculated from the vehicle geometry and kinematics. This is the path the vehicle would take without side slip.

(37)

(38)

(39)

where . is the angle of the vehicle’s gravity center. The actual position and yaw angle are calculated with

(40)

(41)

(42)

This is the actual path the vehicle takes once slip is considered, as the circumferential and lateral forces are included in the calculation of and .

###### Gripping Utilization Factor

The gripping force utilization factor,, characterize the gripping potential of the drive wheels in the presence of the circumferential and lateral forces of the wheels and thus estimate stability of the wheels and vehicle [1].

(43)

At lower values, the wheels have higher gripping potential. Lateral skid and stability loss is usually assumed to occur at > 0.5 [1].

##### 2.4 Hybrid-Electric Powertrain

Figure 12 is a diagram of the vehicle’s hybrid electric powertrain, showing mechanical and electrical links between components. The vehicle model is a series hybrid. This means that power to drive the vehicle’s wheels is provided solely by an electric motor, while the engine is used to produce electrical power (by turning a generator) which charges the battery. From the diagram, it can be seen why it is called a “series” hybrid, as power flows in a line from the engine to the generator and battery and then from the motor to the driveline.



Figure 12: Series hybrid configuration

Figure 13 is the block diagram of the hybrid electric powertrain, which includes the motor, generator, battery, engine, and engine controller.

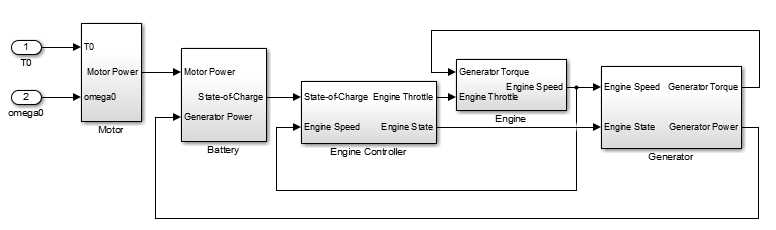


Figure 13: Hybrid electric powertrain model

###### Electric Motor

The electric motor is the source of the driving torque to the wheels. The motor draws energy from the battery, but can also restore energy when it is being used as a brake (the torque is negative). In regenerative braking, a feature of hybrid electric vehicles, the motor acts as a generator when braking and recovering some of the energy used. The motor model subsystem determines power drawn by the motor from the battery. The motor characteristics are given in lookup tables of the motor’s max torque vs. speed and its efficiency vs. torque and speed. The motor has fixed transmission gear ratio between the motor and transfer case input shaft (see Fig. 12). Equations (44) and (45) are the required torque and speed of the motor.

(44)

(45)

where is the mechanical transmission efficiency, is the input torque to the transfer case, the angular velocity of the transfer case input shaft, and is the gear ratio between the transfer case and motor. The required torque to the input shaft is compared to the maximum torque the motor is capable of at its current speed. Since mechanical power of a rotating system is equal to angular speed times torque, the motor’s mechanical power is calculated from motor torque multiplied by its speed. Motor electrical power is calculated by multiplying or dividing mechanical power by the motor’s efficiency, depending on whether electrical power is input or output (whether it is being used as a motor or regenerative brake) [3]. Equation (46) is the motor power equation.

(46)

where is motor input power, is motor output power, and is the motor efficiency from the lookup table as a function of torque and speed.

###### Engine/Generator Set

The engine/generator set is used to recharge the battery and to provide power to the motor while the battery is charging. The engine controller uses the series hybrid thermostat strategy. In this strategy, the engine is activated to recharge the battery when it reaches a minimum level of charge; it is then run until it returns to its maximum limit of charge [4]. Using this strategy, the controller holds the engine at a constant speed. The engine is modeled with lookup tables of maximum torque vs. speed and closed throttle torque vs. speed (closed throttle torque is the torque when the throttle is at zero). Output torque is the maximum torque multiplied by the engine throttle percentage, . The change in engine speed is calculated with the rotational dynamics (equation (47)) using the engine’s supplied torque and torque load from the generator where is the inertia [5]:

(47)

Equation (47) is solved for , then used in an integral to calculate .

The generator subsystem is structured using the same lookup tables as the motor. The generator torque is given by equation (48).

(48)

where generator torque is the maximum torque, a function of the generator speed , multiplied by engine throttle . Equation (49) is the generator speed

(49)

where is the gear ratio between the generator and engine. The generator power is

(50)

where is the generator efficiency.

The engine controller adjusts the engine output to keep the battery charging current constant and determines whether the engine should be running based on the battery’s current charge level (SOC). There are four states for the engine controller:

1. SOC is below minimum, in which case the engine is always turned on.
2. SOC is above maximum, in which case the engine is always turned off.
3. SOC is between max and min and was off in the previous step of the simulation, in which case it is left off.
4. SOC is between max and min and the engine was in use in the previous step, in which case it is left on.

When the engine is started, the generator provides a cranking torque to start the engine. Once the engine is up to speed, it turns the generator to provide power to charge the battery [3].

###### Battery

The battery subsystem models the charging and discharging of a lithium ion battery. It calculates the variation in charge level of the battery from all the electrical sources drawing power from or contributing power to it. Since electrical power is voltage times current, power from the motor and generator is summed and then divided by the battery’s voltage to determine the current into or out of the battery. The battery’s state of charge () measures the charge level of the battery, between zero (empty) and one (fully charged). is calculated using the integral of the battery current:

(51)

is the sum of all currents charging or drawing from the battery, positive if charging and negative if draining; Is the battery efficiency. is the battery’s charge capacity in Ampere-hours, and the charge level at the start of the simulation. An Ampere-hour measures the amount of charge that a battery will supply with a current of 1 A in one hour. The battery’s voltage is a function of its state of charge [5]. As the battery depletes, there will be some loss of voltage. A table gives the battery’s voltage vs. its SOC for a lithium ion battery.

# References

[1] Andreev, A.F., Kabanau, V.I., and Vantsevich, V.V., *Driveline Systems of Ground Vehicles: Theory and Design*. V.V. Vantsevich, Scientific and Engineering Editor, Taylor and Francis Group/CRC Press, ISBN 978-1-4398-1727-8, 2010.

[2] V. V. Vantsevich, “AWD Vehicle Dynamics and Energy Efficiency Improvement by Means of Interaxle Driveline and Steering Active Fusion”, ASME 15th International Conference on Advanced Vehicle Technologies, Portland, OR, August 4-7, 2013.

[3] Liu, W., 2013, *Introduction to Hybrid Vehicle System Modeling and Control*, Wiley, Hoboken, NJ, pp. 25-61.

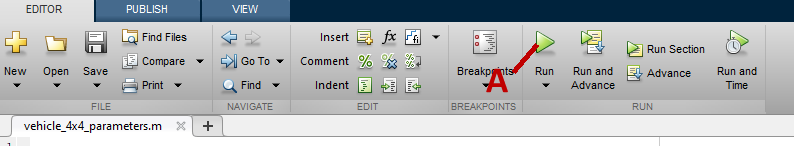
[4] Ehsani, M., Gao, G., and Emadi, A., 2010, *Modern Electric, Hybrid Electric, And Fuel Cell Vehicles, Fundamentals, Theory, And Design*. 2nd ed., CRC Press, Boca Raton, FL, pp. 257-258.

[5] Kanber, B., and Baglione, M., 2011, "Developing an Extensible and Concise Simulink Toolset for Hybrid Vehicle Modeling and Simulation," SAE Paper No.2011-01-0755.

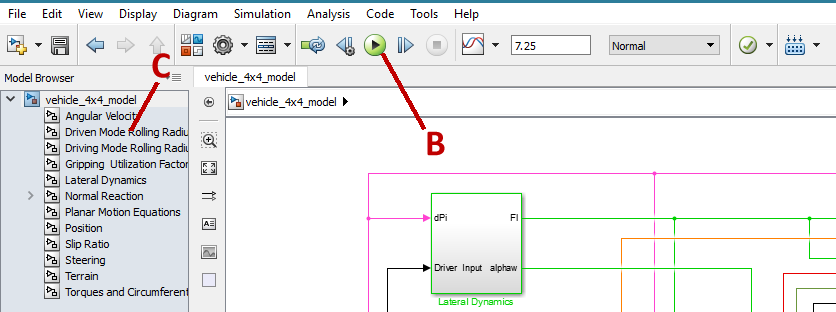
## Assignment:

Instructions for performing the virtual test of the vehicle are as follows:

1. Necessary parameters for the vehicle are contained in the script 4x4\_vehicle\_paramters.m. Values of the parameters may be changed in this script, and run with the run button (A) to load the values to the workspace.



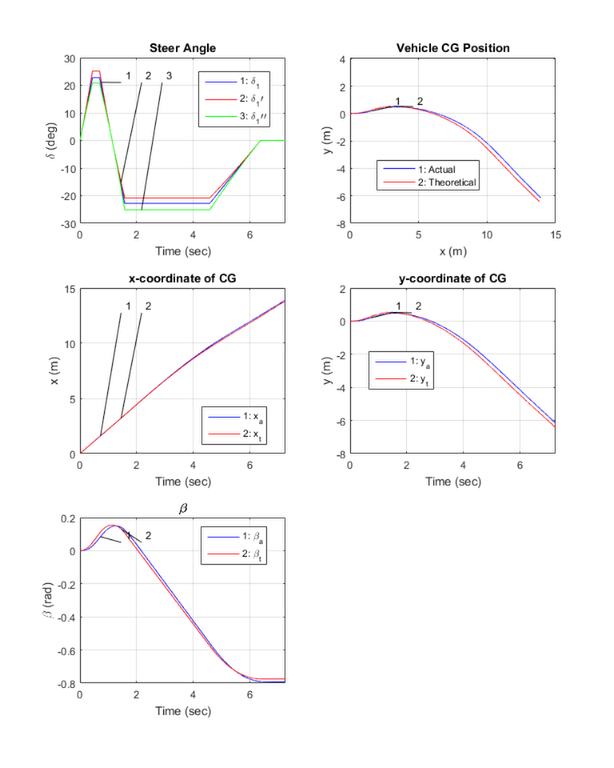
1. After parameters are loaded, the model vehicle\_4x4\_model.slx may be run using the run button (B). The contents of individual subsystems may be browsed by double-clicking the subsystem, or using the menu (C).

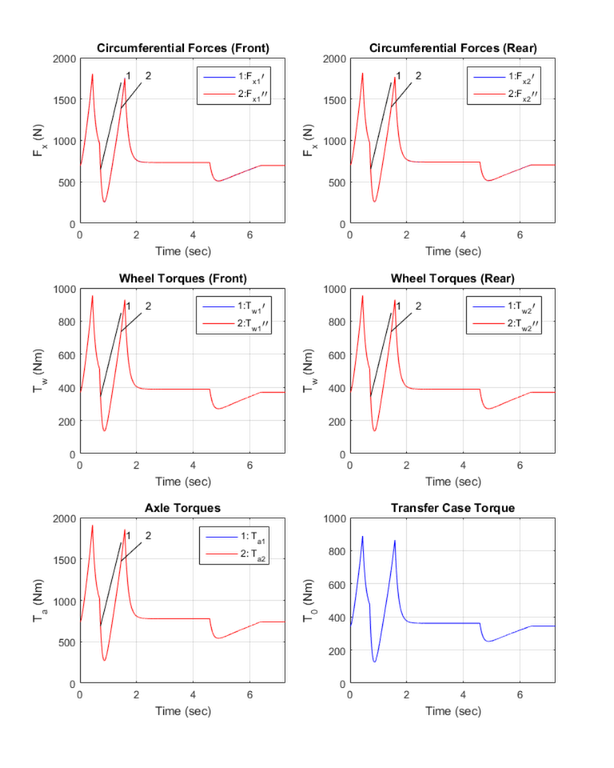


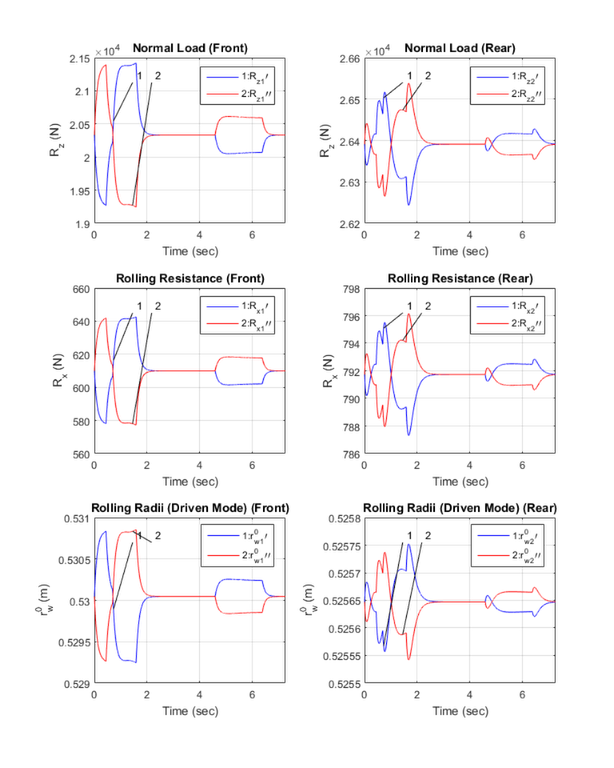
1. When the model is run, data is saved to the workspace; the list of saved variables is shown in the workspace view of the main MATLAB window (D). Plots of the results may be created using these variables (see Section 1.3).

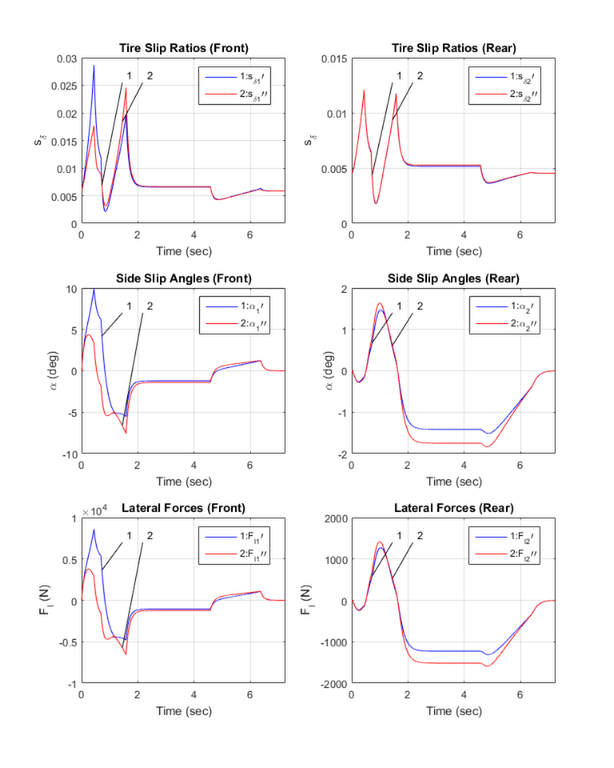
## 

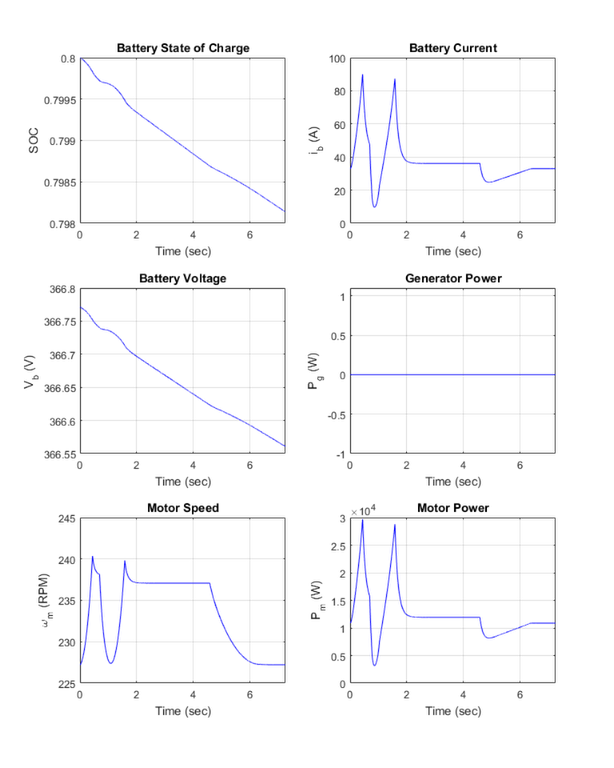
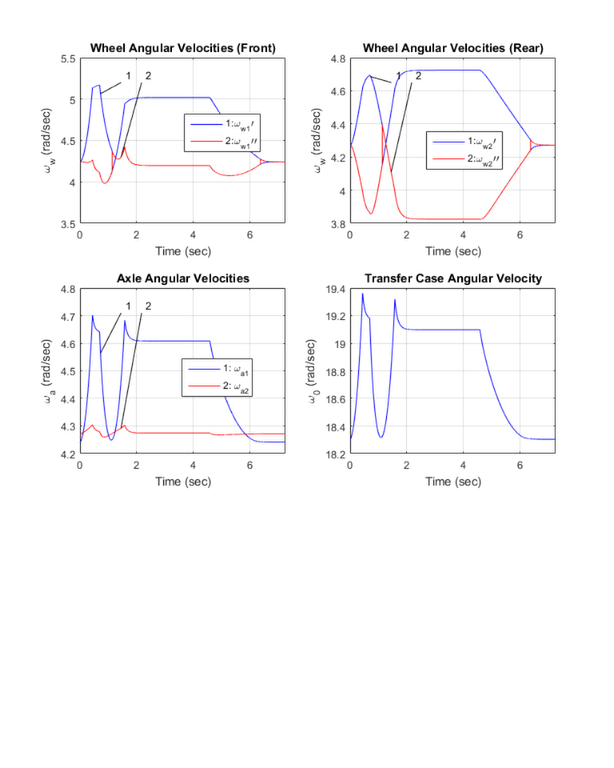
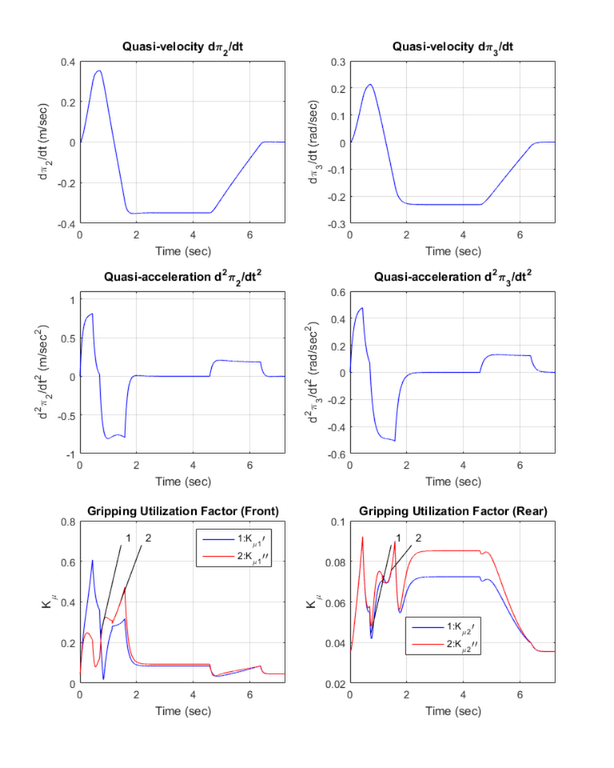
## Test Results

****

****

****

****

****