Building College-University Partnerships for Nanotechnology Workforce Development

Understanding Variation and Statistical Process Control: Variation and Process Capability Calculations

Outline

- Variation and It's Relationship to Quality
- The Normal Distribution as a Model for Describing Variation
- Comparing Variation to Specifications
- Relationship between Cp and Cpk
- Calculation of Cpk and Ppk

New Face of Quality – Low Variation

Quality = possessing one or more functional characteristics that improve the performance of a product or service. i3 vs i5 has lower # of cores, low cache memory and no Turbo Boost for speed.

CORMETECH

Michael Close is developing an addition process to adjust each batch of incoming raw material to the same acetate content before being released into production. -- I don't care where they set it, I just wish it would stay the same! --

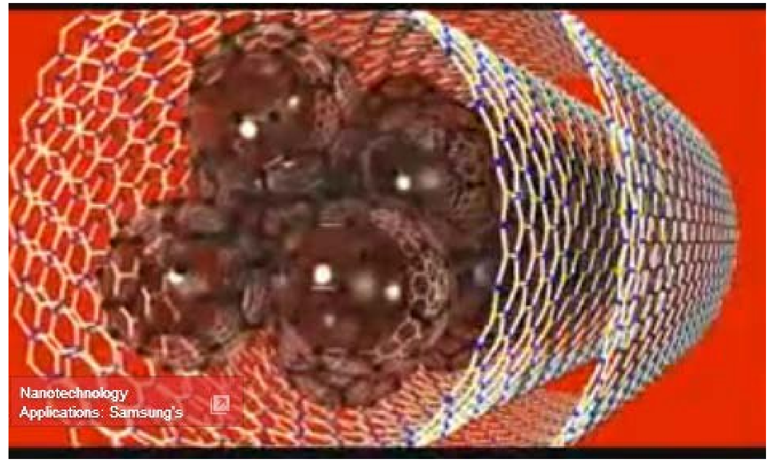
HELIX Material Solutions

Carbon Nanotubes are a true example of nanotechnology, embodying a unique combination of electrical, thermal, and structural properties. Helix's state-of-the-art chemical vapor deposition (CVD) process enables the production of nanotubes with controlled *Diameter* (**1.2 ~ 1.5 nm diameter)** which is critical to those physical properties.

Data from the Process Provides Variation Insight

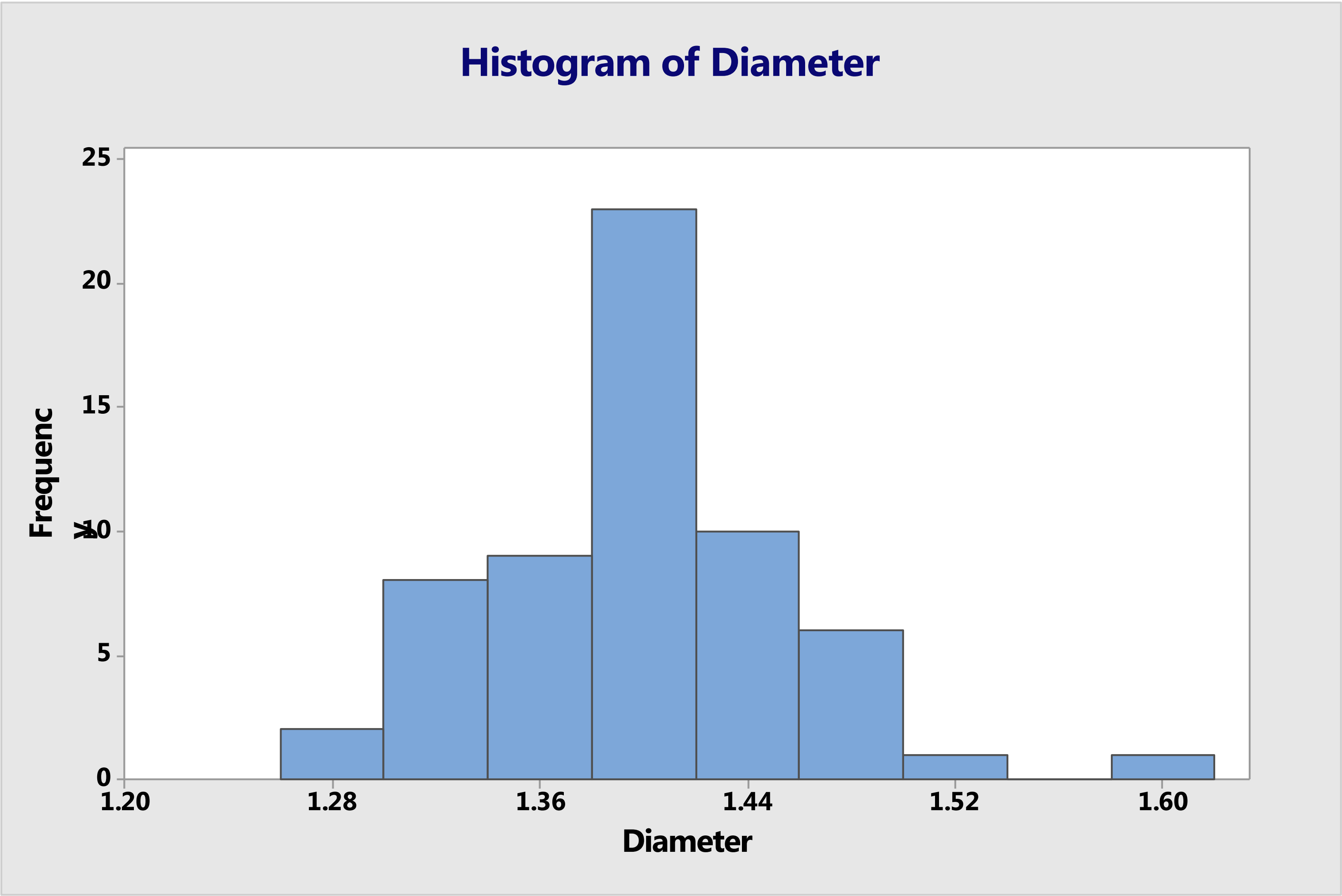
Worksheet 1 ***

↓	C1	C2-D	C3
	Sample	Date/Time	Diameter
1	1	10/19 10:07	1.48
2	2	10/19 10:08	1.36
3	3	10/19 10:14	1.31
4	4	10/19 10:17	1.30
5	5	10/19 10:18	1.41
6	6	10/19 10:20	1.33
7	7	10/19 10:22	1.33
8	8	10/19 10:25	1.32
9	9	10/19 10:26	1.40
10	10	10/19 10:27	1.40
11	11	10/19 10:30	1.41
12	12	10/19 10:33	1.40
13	13	10/19 10:35	1.36
14	14	10/19 10:36	1.39
15	15	10/19 10:44	1.32
16	16	10/19 10:47	1.41
17	17	10/19 10:49	1.46
18	18	10/19 10:52	1.35
19	19	10/19 10:56	1.40
20	20	10/19 10:58	1.42

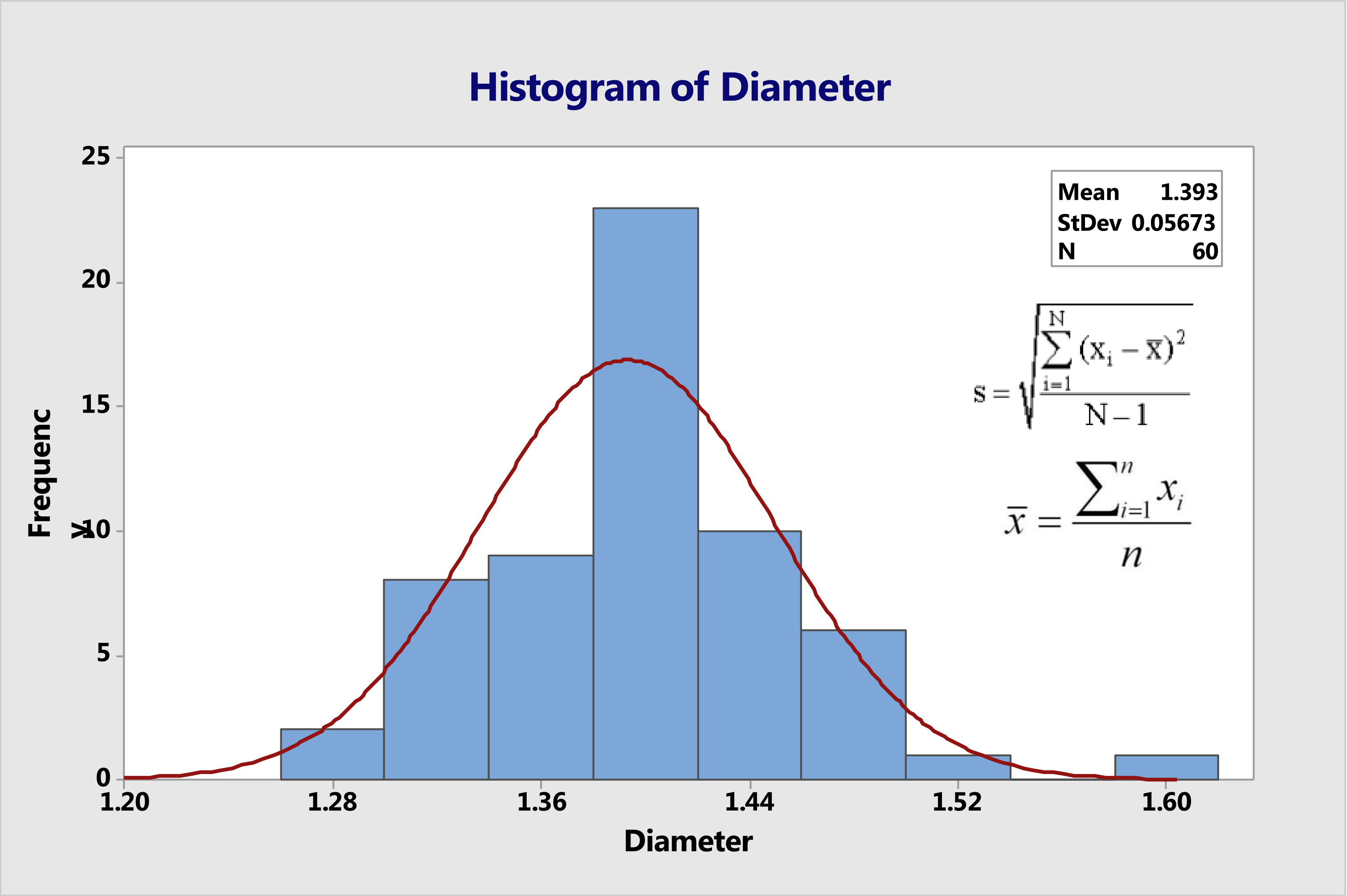


Since the physical properties of carbon nanotubes are a function of it's volume / surface area ratio, the diameter of the nanotubes is a key quality characteristic.

A Histogram Outlines the Diameter Distribution



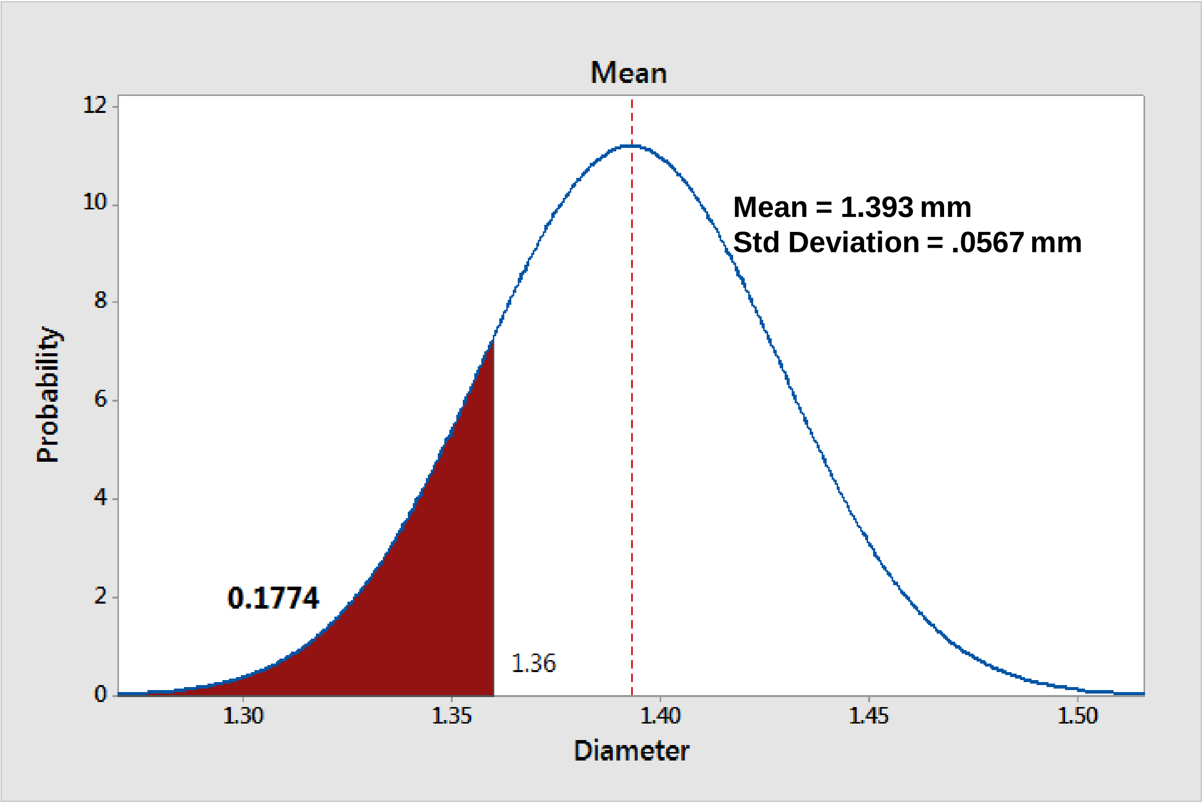
A Normal Distribution defines the Diameter Variation



$$s = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N-1}}$$

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Data Assumed as a Sample of a Normal Population



$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$$

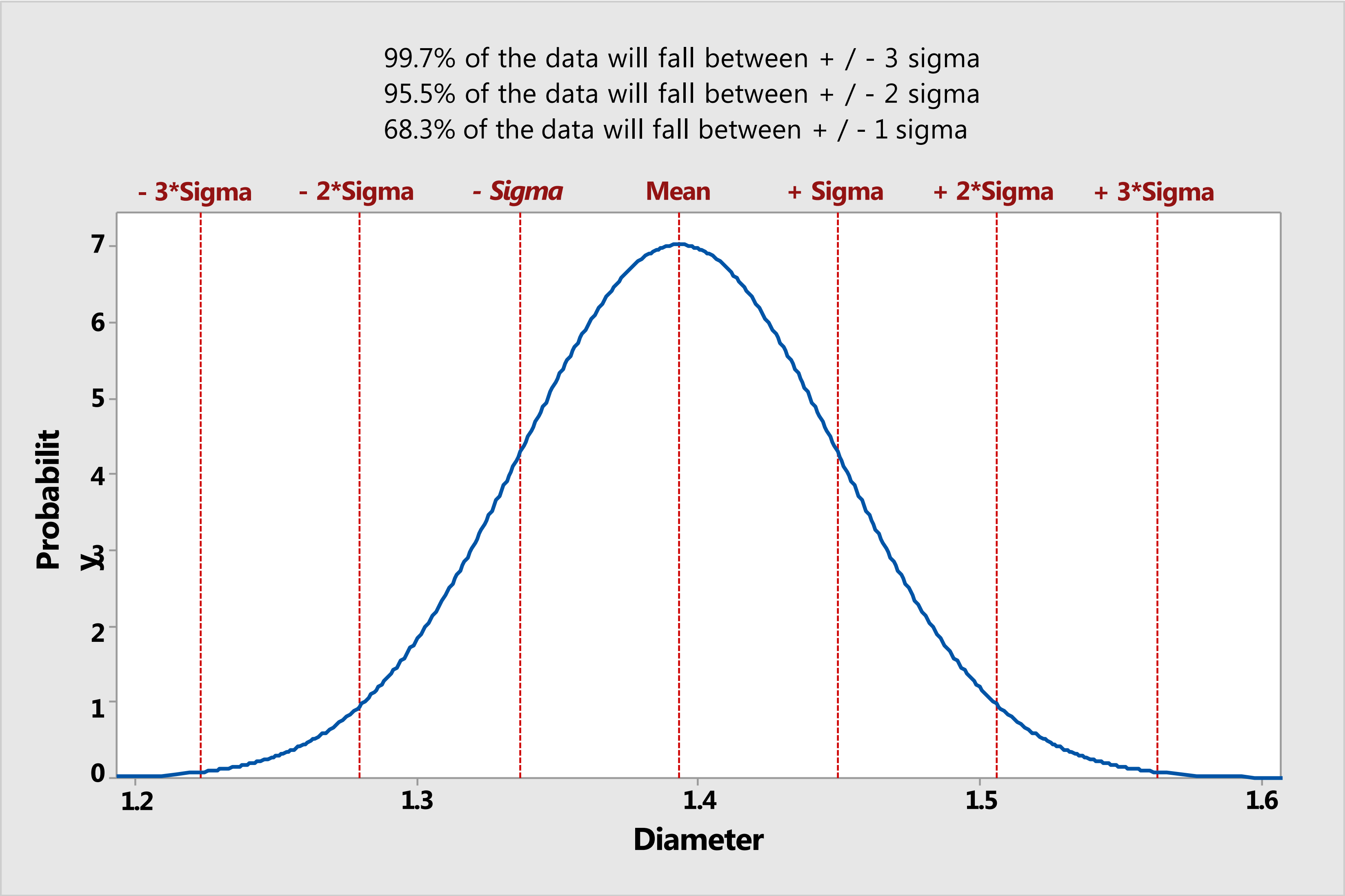
$\mu = \textit{Mean}$

$\sigma = \textit{Standard Deviation}$

Probability of a nanotube diameter of 1.36 or less is about 18%.

Assuming a normal population distribution allows calculation of probabilities of interest.

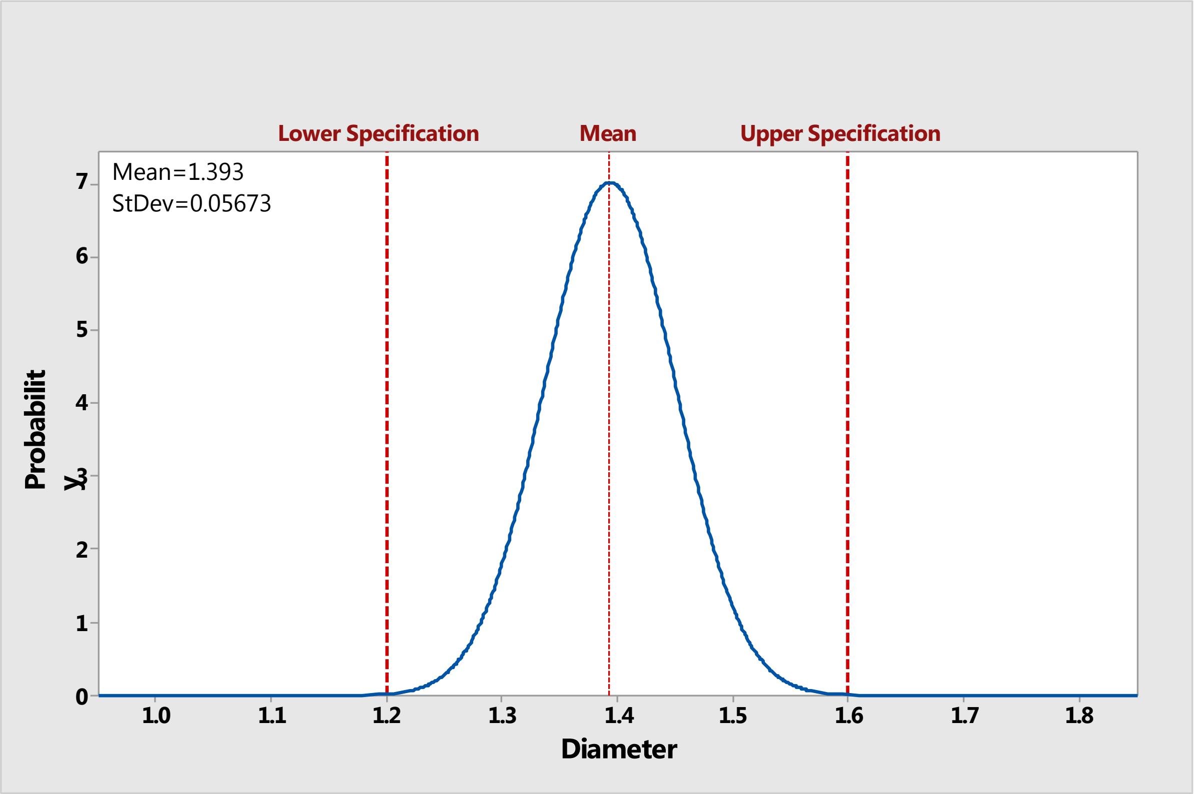
Assuming a Normal Distribution Establishes Limits



Cp to Measure Process Capability

Is my process capable of meeting my customers specs?

$$Cp = \frac{(\text{Upper Spec} - \text{Lower Spec})}{6 * \text{Standard Deviation}}$$



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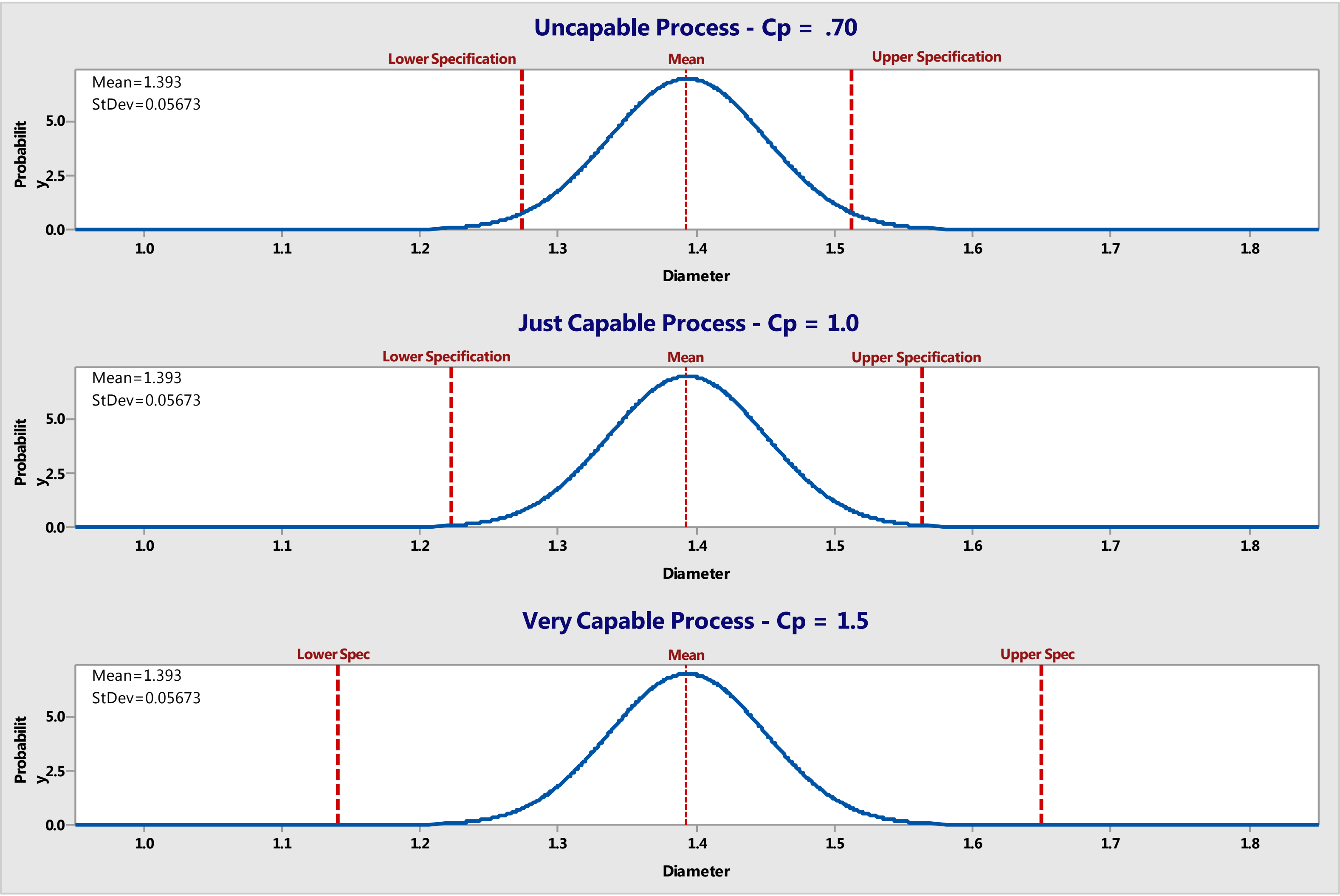

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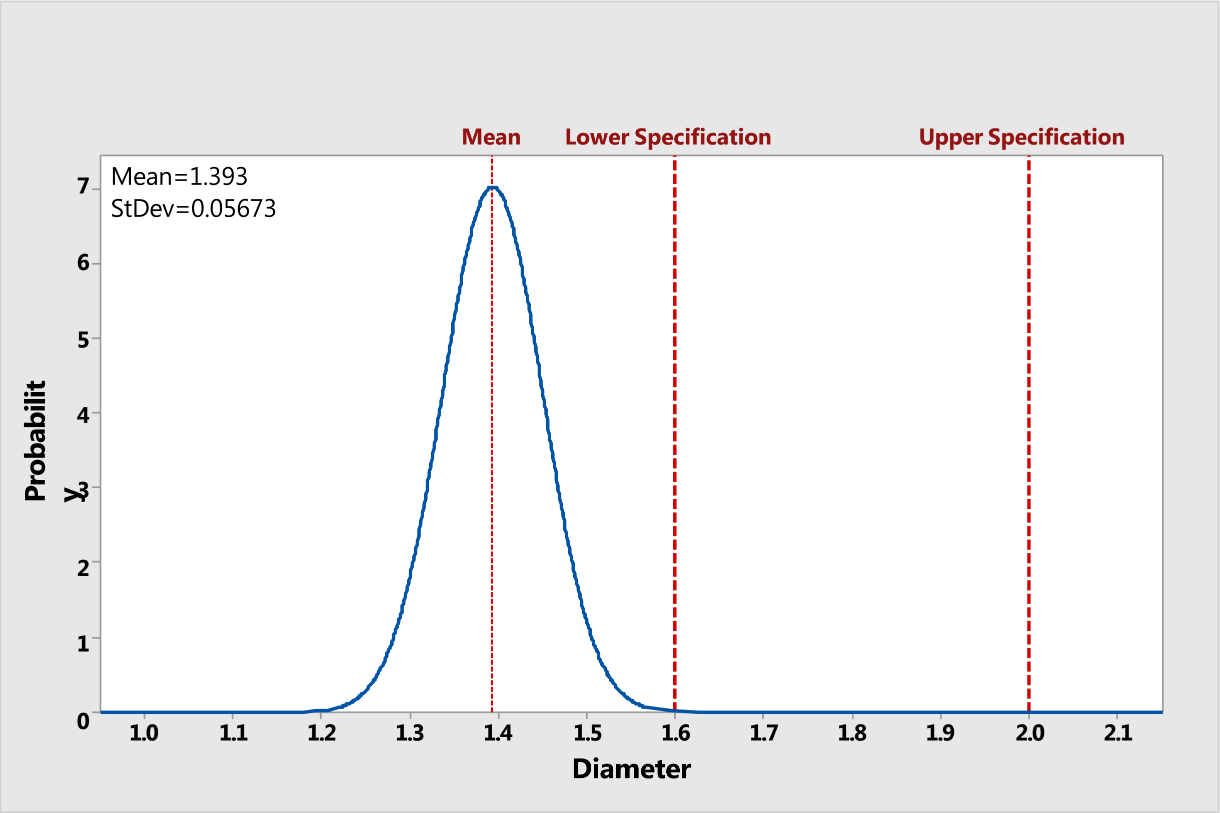
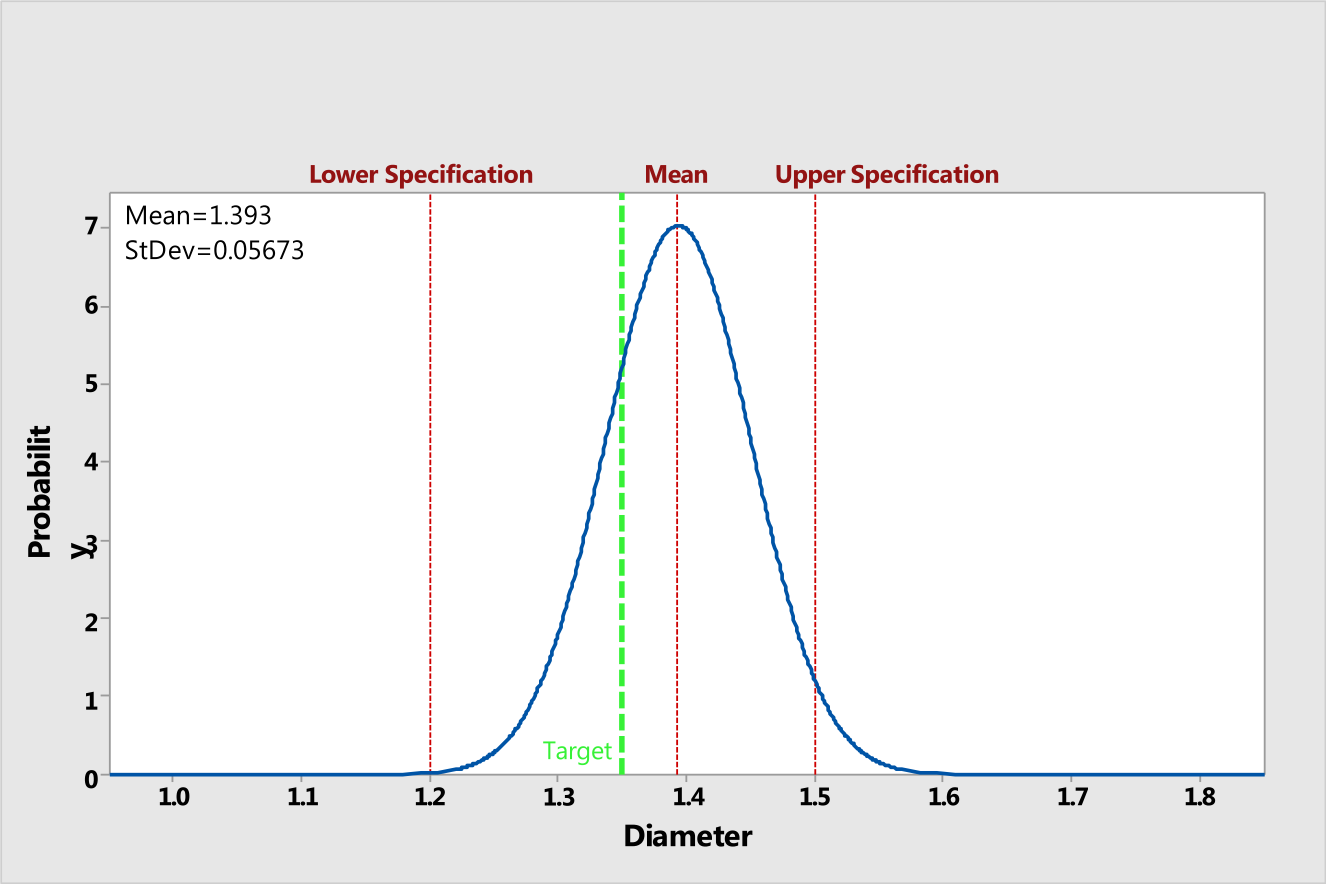


Process Capability

$$Cpk = \frac{\min\left[(\text{Upper Spec} - \text{Avg}) \text{ or } (\text{Avg} - \text{Lower Spec})\right]}{3 * \text{Standard Deviation}}$$



Process Capability Calculations

$$Cp = \frac{(\text{Upper Spec} - \text{Lower Spec})}{\text{Process Variation}}$$

$$Cpk = \frac{\min\left[(\text{Upper Spec} - \text{Avg}) \text{ or } (\text{Avg} - \text{Lower Spec})\right]}{\text{Half the Process Variation}}$$

Mean=1.393

StDev=0.05673

Spec Range = 1.2 - 1.5 nm

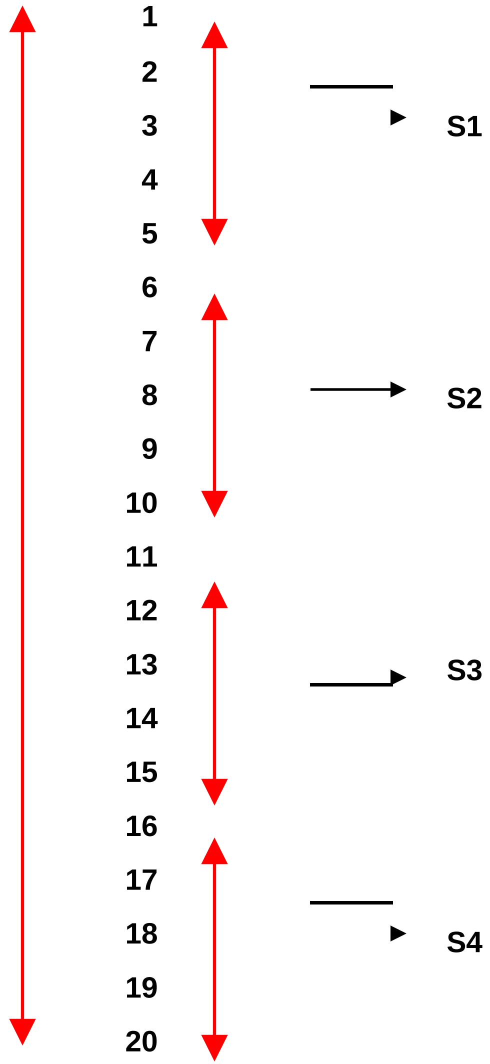
Cp = (1.5 – 1.2) / (6 x .0567) = .88

Cpk = (1.5 – 1.393) / (3 x .0567) = .63

Process Capability
(Short-term and Long-term)

Std Dev Overall

Std Dev Long-term



Std Dev Short-term =
Std Dev Pooled =
Std Dev (within) =

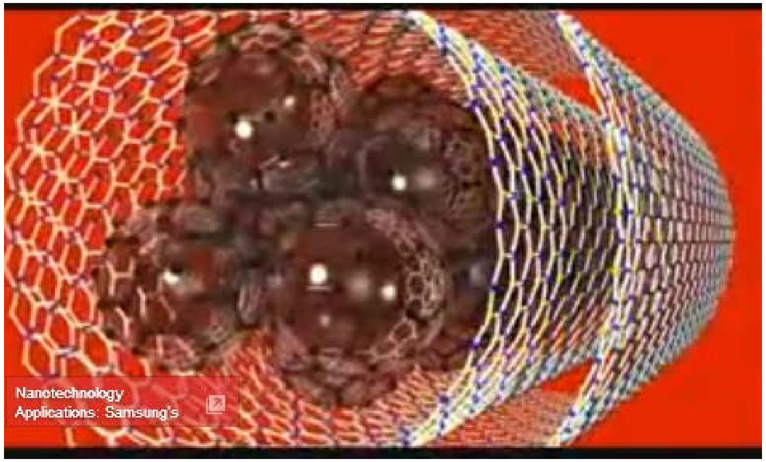
$$[(S1**2 + S2**2 + S3**2 + S4**2) / 4]^{.5}$$

Ppk & Pp are Cpk & Cp with Std Dev = Std Dev Overall

Data for Long-term Process Capability

Diameter Dataset over Time ***

↓	C1	C2-D	C3
	Subgroup	Date/Time	Diameter
1	1	10/13 8am	1.4595
2	1	10/13 8am	1.4350
3	1	10/13 8am	1.4105
4	1	10/13 8am	1.4210
5	1	10/13 8am	1.4175
6	2	10/13 12pm	1.3615
7	2	10/13 12pm	1.4105
8	2	10/13 12pm	1.4630
9	2	10/13 12pm	1.4280
10	2	10/13 12pm	1.3090
11	3	10/13 4pm	1.3860
12	3	10/13 4pm	1.3090
13	3	10/13 4pm	1.3720
14	3	10/13 4pm	1.3545
15	3	10/13 4pm	1.3790
16	4	10/13 8pm	1.3720
17	4	10/13 8pm	1.4420
18	4	10/13 8pm	1.4000
19	4	10/13 8pm	1.4175
20	4	10/13 8pm	1.4035

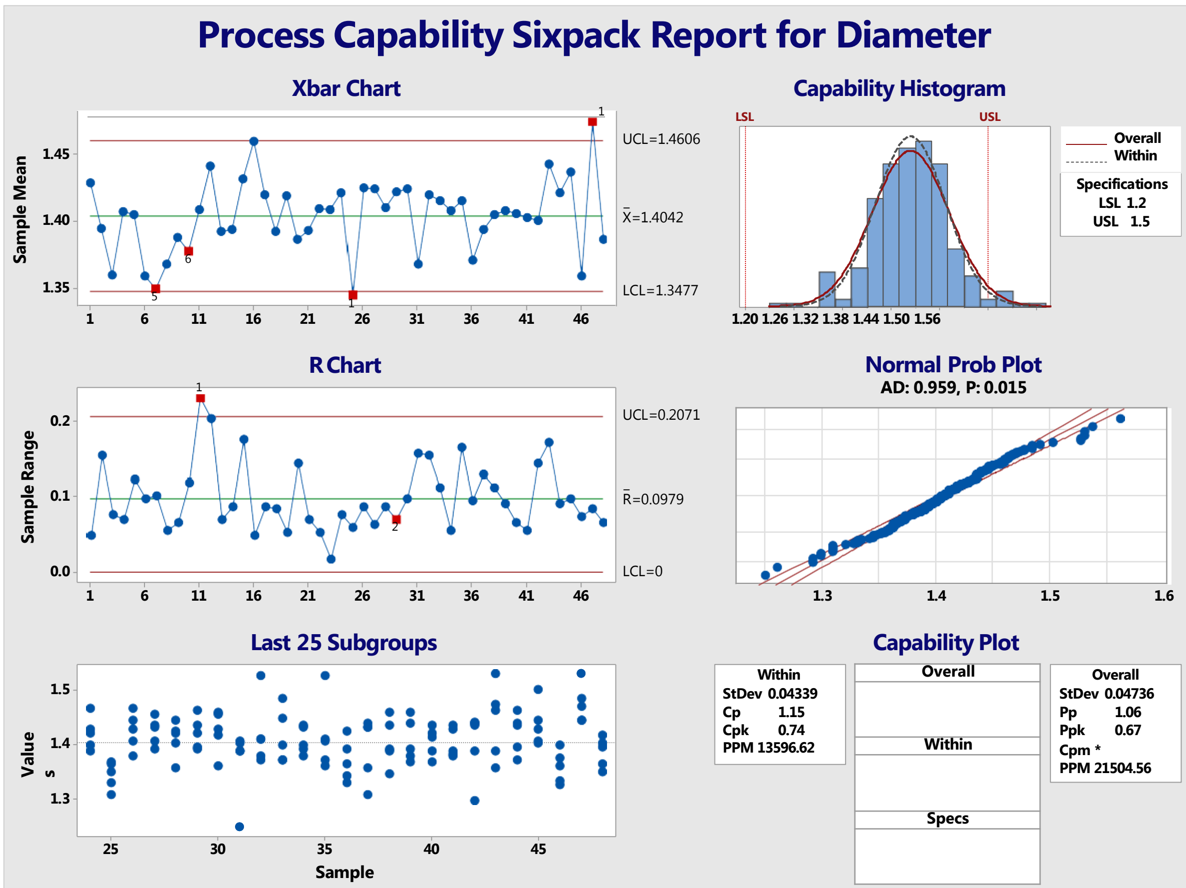


Since the physical properties of carbon nanotubes are a function of it's volume / surface area ratio, the diameter of the nanotubes is a key quality characteristic.

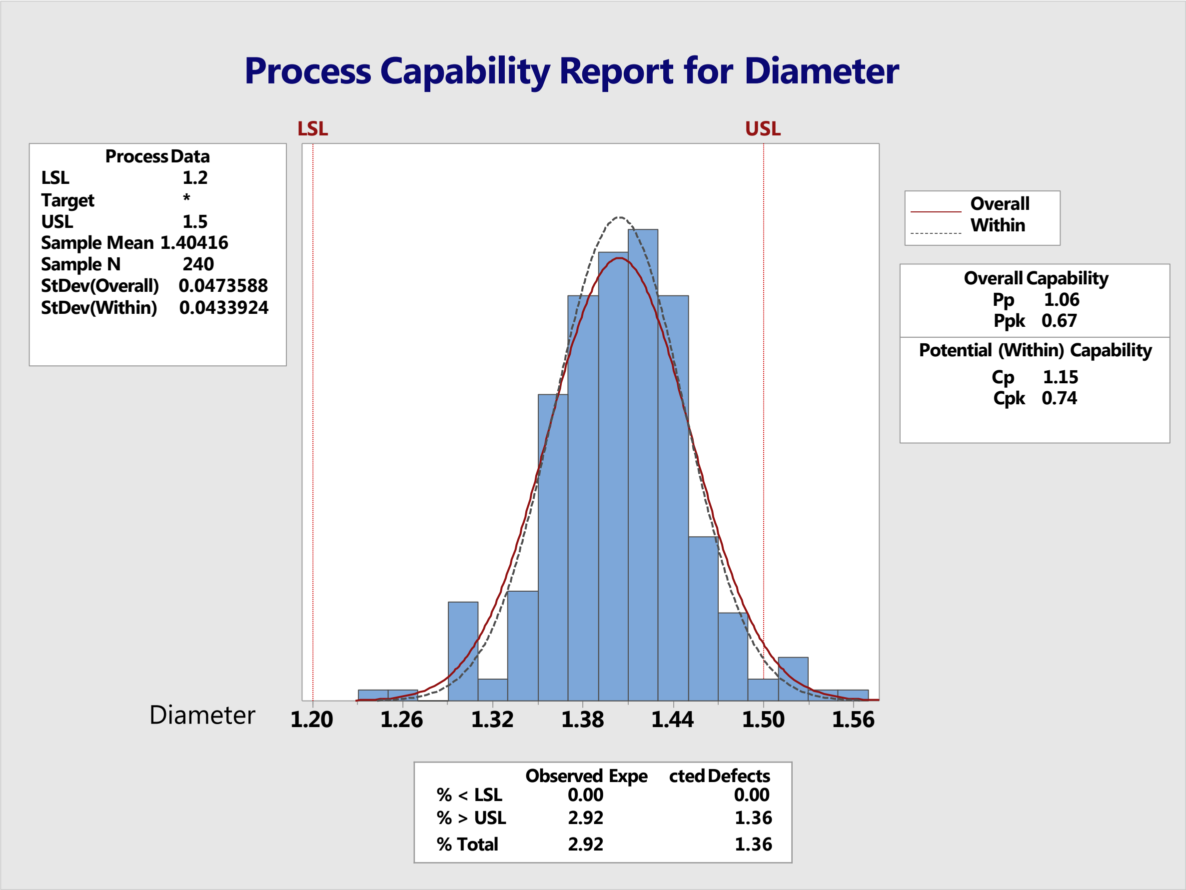
Calculate the Process Capability

Calculating long-term / short-term process capability requires;

1. Upper and/ or lower specifications.
2. Confirmation that the process is in control.
3. Confirmation that the process data is normally distributed.
4. Process data representing long-term & short-term variation.



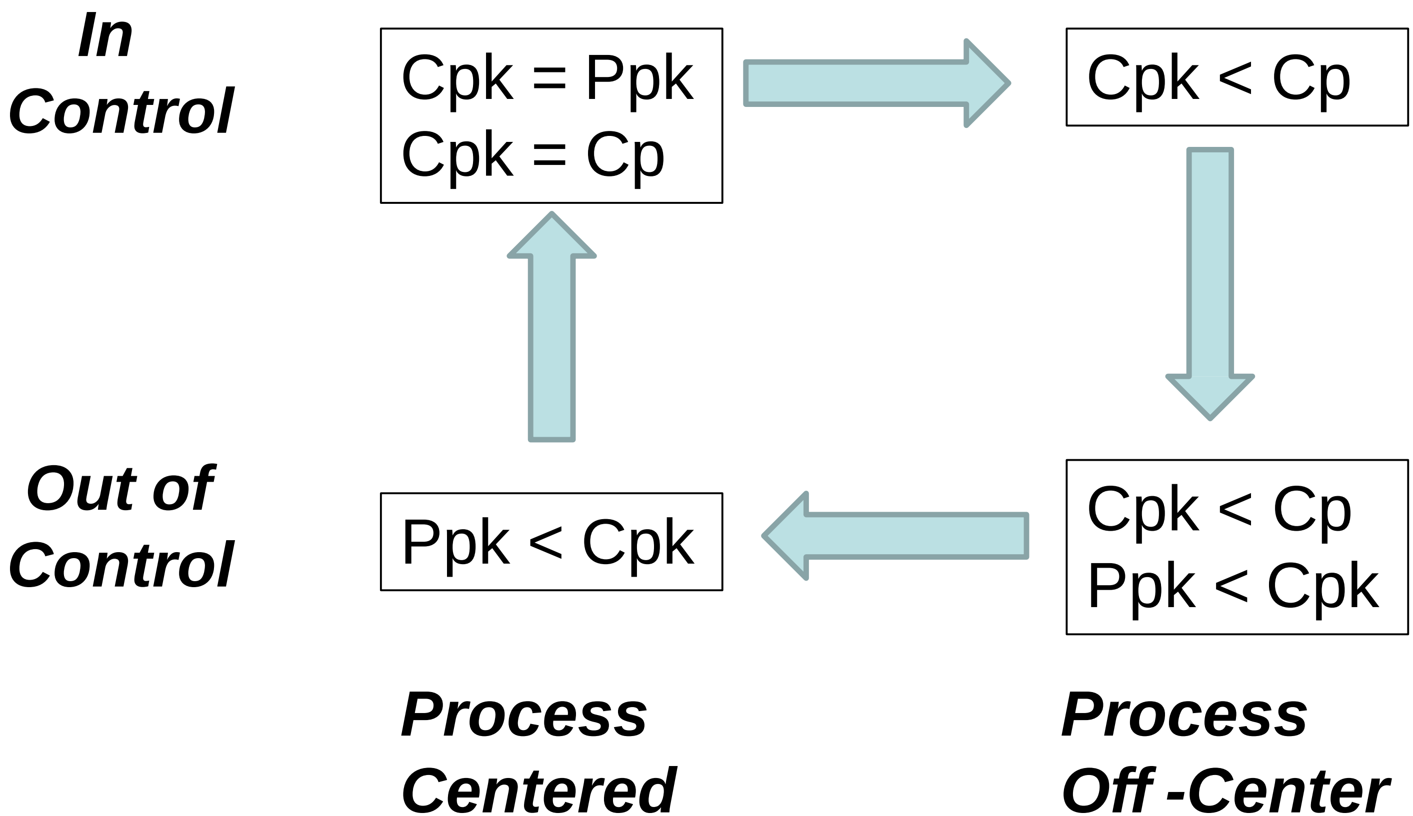
Interpretation of Capability Output



Use the Capability Analysis to reach the following conclusions;

1. Process variation is about equal to the specs (Pp is about = 1.0).
2. Process is off center (Ppk < Pp).
3. Process is fairly in control (Ppk is about = Cpk).

Relationship Between Cpk vs Ppk vs Cp



Acceptable Cpk Levels

Defect Levels Associated with Cpk Values

Cpk	% Defect	Cpk	% Defect
0.00	50.000	0.80	0.820
0.20	27.425	1.00	0.135
0.40	11.507	1.20	0.016
0.60	3.593	1.33	0.003

Airline Industry - Safety Cpk = 2 or more
Baggage Cpk = .4 or less

Pharmaceutical Industry - Cpk = 1.75
Auto Industry - Cpk = 1.33
Typical Manufacturing - Cpk = 1.00

Nanoscale Manufacturing = ?

Conclusions

- Modern nanoscale processes seeking to produce high Quality products will seek to understand and minimize the variation in their key Quality characteristics.
- Graphical analysis such as the histogram and normal probability plots allow one to understand and model the variability in process outputs.
- Capability indices such as Cp and Cpk compare the process variation to the customers specifications. They are used to determine if the variation is wider than the specifications, the process is not centered or both.
- If process data is collected over a long time period, Pp and Ppk are used to determine the long-term process performance capability.